

Math-251: Linear Algebra Week 1

Book: Elementary linear Algebra (Tenth Edition)
Howard Anton

Chapter 1: Systems of linear Equations and Matrices

Lecture 1 (Week 1)

(1) 1.1 Introduction to system of linear Equations

(2) 1.2 Gaussian Elimination

(3) 1.3 Matrices and Matrix operation

1.1 Introduction

In 2 dimension a line can be represented by
 $ax + by = c$ where $a, b \neq 0$

In 3D a plane can be represented by an equation
 $ax + by + cz = d$ where $a, b, c \neq 0$

There are 3 ~~ways~~ ways to solve the system of linear equation

(A) Graphically

(B) Algebraically using add or sub

(C) Algebraically using substitution.

(A)

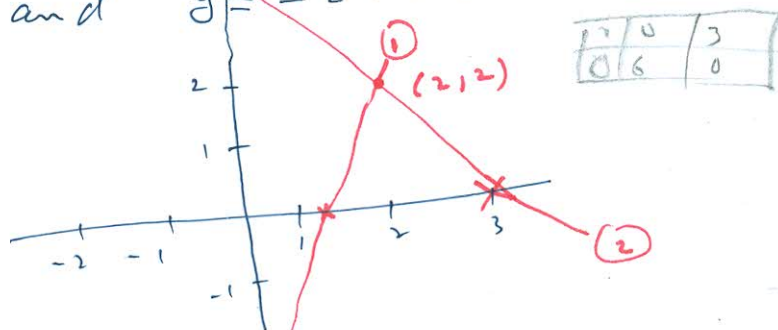
Graphically

Let us have two lines

$$y = 3x - 4 \quad (1)$$

$$y = -2x + 6 \quad (2)$$

x	0	4/3
y	-4	0



1	0	3
0	6	0

(B) Now solve (1) & (2) Algebraically
or we can write

$$3x - y = 4 \quad \text{--- (1A)}$$

$$2x + y = 6 \quad \text{--- (2)}$$

Solve (1A) & (2A)

$$\begin{array}{r} 3x - y = 4 \\ 2x + y = 6 \\ \hline \end{array}$$

$$x = -2$$

$$x = 10$$

$$x = \frac{10}{5} = 2$$

put in (1A) $3 \times 2 - y = 4$
 $6 - y = 4$
 $6 - 4 = y$
 $2 = y$

Note: (1) If the lines cross once, there will be one solution



(2) If the lines are parallel, there will be no solution.

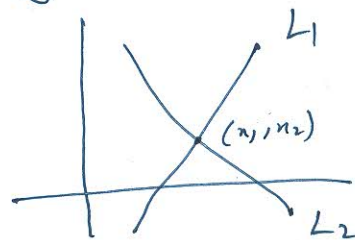
(3) ~~→~~

Systems of Equations

If there are two straight lines L_1 and L_2 Then

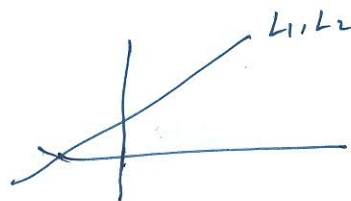
1- If L_1 and L_2 intersect at exactly one point

Then there is a unique solution



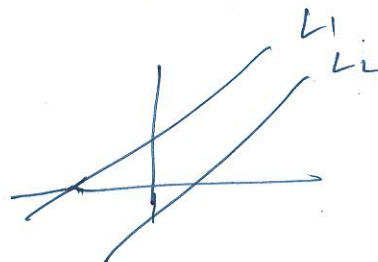
2- If L_1 and L_2 are coincident

Then there are infinitely many solutions



3- If L_1 and L_2 are parallel

Then there is no solution



Solve the linear equation

$$\begin{aligned} x - y &= 1 & \text{---(1)} \\ 2x + y &= 6 & \text{---(2)} \end{aligned}$$

Sol:

add (1) & (2)

$$\begin{array}{r} x - y = 1 \\ 2x + y = 6 \\ \hline 3x = 7 \\ x = \frac{7}{3} \end{array}$$

put in (1) $\frac{7}{3} - y = 1 \Rightarrow y = \frac{7}{3} - 1 = \frac{4}{3} //$

There is a unique solution $(\frac{7}{3}, \frac{4}{3})$

Ex 2: Solve the linear system

$$\begin{aligned} x + y &= 4 & \text{---(1)} \\ 3x + 3y &= 6 & \text{---(2)} \end{aligned}$$

Solve:

we can write

$$\begin{array}{r} \times \text{ by } 3 \text{ eqn (1)} \\ 3x + 3y = 12 \\ 3x + 3y = 6 \\ \hline 0 - 0 = 6 \end{array}$$

$$0 = -6$$

So the given system has no solution.

Ex 3: Solve $4x - 2y = 1$ --- (1) \times by 4
 $16x - 8y = 4$ --- (2)

Soln:

$$\begin{array}{r} 16x - 8y = 4 \\ 16x - 8y = 4 \\ \hline 0 = 0 \end{array}$$

This means there are infinitely many solutions in this case we choose any equation and express one variable in the form of others. here x.

Let eqn (1) be

$$4x - 2y = 1 \quad \text{or} \quad 4x = 1 + 2y \quad \text{or} \quad x = \frac{1}{4} + \frac{1}{2}y \quad \text{---(3)}$$

Now assign any arbitrary value to y, $y = t$ (parameter) Now
 $x = \frac{1}{4} + \frac{1}{2}t$ --- (4) Now we put any value of t, we get different value of x, let suppose we put $t = 0$ in (4) then $x = \frac{1}{4}$ so $(\frac{1}{4}, 0)$ is the solution
let $t = 1$ in (4) then $x = \frac{3}{4}$ so $(\frac{3}{4}, 1)$ is another solution and so on.

1.2 Gaussian Elimination

Q₁ Use Gauss Elimination to solve the system of linear eqs

$$\begin{aligned}x_1 + 5x_2 &= 7 & \text{--- (1)} \\ -2x_1 - 7x_2 &= -5 & \text{--- (2)}\end{aligned}$$

Soln: Write the Augmented matrix of eqns (1) & (2)

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array} \right]$$

Now using the Elementary Row operations convert it into Row Echelon form

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array} \right] R_2 \rightarrow R_2 + 2R_1$$

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array} \right] R_2 \rightarrow \frac{R_2}{3}$$

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array} \right] R_1 \rightarrow R_1 - 5R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array} \right]$$

This is a Row Echelon form. Now the value of
 $x_1 = -8$ and $x_2 = 3$

Q₂ solve the system of linear equations

$$\begin{aligned}x - 3y + z &= 4 \\ 2x - 8y + 8z &= -2 \\ -6x + 3y - 15z &= 9\end{aligned}$$

Soln:

The Augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{array} \right] \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 + 6R_1 \end{array}$$

~~$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ 0 & -5 & -3 & 11 \end{array} \right]$$~~

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ 0 & -15 & -9 & 33 \end{array} \right] \begin{array}{l} R_2 \rightarrow \frac{1}{2} R_2 \\ R_3 \rightarrow R_3 + 15R_2 \end{array}$$

~~$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & -3 & 5 \\ 0 & -15 & -9 & 33 \end{array} \right] \begin{array}{l} R_3 \rightarrow R_3 + 15R_2 \end{array}$$~~

~~$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & +1 & -3 & +5 \\ 0 & 0 & -54 & -54 \end{array} \right] \begin{array}{l} R_3 \rightarrow \frac{R_3}{-54} \end{array}$$~~

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & +1 & -3 & +5 \\ 0 & 0 & +1 & -2 \end{array} \right]$$

Now using the Back substitution we can find the value of

x, y, z

here ~~z = 2~~
z = -2

~~$$\begin{array}{l} y - 3z = 5 \\ y + 6 = 5 \implies y = -1 \\ y - 3z = 5 \\ y = 1 \\ -y + 6 = 5 \\ -y = -1 \implies y = 1 \\ y = 1 \end{array}$$~~

$$\begin{array}{l} x - 3y + z = \\ x + 3 + (-1) \\ x + 1 = 4 \\ x = 3 \end{array}$$

