

 

**3. Show that the map *T* : R 2 *→* R 2 given by *T*(*x, y*) = (*x* + *y, x − y*) is linear.**

T(x,y) = (x+y , x-y) ……… 1

If a = (a1,a2) and b = (b1,b2)

We have to show two things to be linear

1. T(a +b) = T(a)+T(b). (additivity.)

2. T(k a) = k T(a). (homogeneity.)

 1. T(a +b) = T(a)+T(b). (additivity.)

 T(a +b) = T((a1,a2) +(b1,b2))

 = T(a1+b1, a2+b2)

 = (a1 + b1+ a2+ b2, a1+b1- a2- b2) from………. 1

 = ( (a1+a2) + (b1+b2) , (a1-a2) + (b1- b2) )

 = (a1+a2 , a1-a2) + (b1+ b2 , b1- b2) ……… 2

 T(a) = T(a1,a2)

 = (a1+a2 , a1-a2) from………. 1

 T(b) = T( b1,b2)

 = (b1+ b2 , b1- b2) from………. 1

 T(a)+T(b) = (a1+a2 , a1-a2) + (b1+ b2 , b1- b2) ……… 3

While 2 = 3 then T(a +b) = T(a)+T(b)

2. T(k a) = k T(a). (homogeneity.)

T(k a) = T(k(a1,a2) )

= T(ka1, ka2)

= (ka1+ ka2, ka1- ka2) from………. 1

= k(a1+ a2, a1- a2)

= kT(a1, a2)

= kT(a)

So T(k a) = k T(a)

 Therefore , T is linear transformations

4. Verify rank-nullity theorem for the linear operator *T* : R 7 *→* R 7 having 4 elements in
the basis of kernel of *T* with 3 elements in the basis of image of *T*.

Rank(T) = dim(range or image of T) = 3

Nullity(T) = dim(kernel of T) = 4

 Dim(domain of T)= n = 7

So 3 + 4 = 7 it is Achieves rank-nullity theorem

5. Find a *LU*-decomposition of the matrix *A* = $\left[\begin{matrix}5&-1\\-1&-1\end{matrix}\right]$

A= $\left[\begin{matrix}5&-1\\-1&-1\end{matrix}\right]$ $\left[\begin{matrix}\*&0\\\*&\*\end{matrix}\right]$

 $\left[\begin{matrix}1&-\frac{1}{5}\\-1&-1\end{matrix}\right]$ $\frac{1}{5}$ R1 $\left[\begin{matrix}5&0\\\*&\*\end{matrix}\right]$

 $\left[\begin{matrix}1&-\frac{1}{5}\\0&-\frac{6}{5}\end{matrix}\right]$ 1\* R1 + R2$\rightarrow $ R2 $ \left[\begin{matrix}5&0\\-1&\*\end{matrix}\right]$

U= $\left[\begin{matrix}1&-\frac{1}{5}\\0&1\end{matrix}\right]$ **-** $\frac{5}{6}$R2  **L=** $\left[\begin{matrix}5&0\\-1&-\frac{6}{5}\end{matrix}\right]$

A = L \* U

$\left[\begin{matrix}5&-1\\-1&-1\end{matrix}\right]$ = $\left[\begin{matrix}5&0\\-1&-\frac{6}{5}\end{matrix}\right]$\*$\left[\begin{matrix}1&-\frac{1}{5}\\0&1\end{matrix}\right]$

6- Find the singular values of the matrix *A* = $\left[\begin{matrix}3&1\\0&3\\1&0\end{matrix}\right]$

The first step is to find the eigenvalues of the matrix

ATA = $\left[\begin{matrix}3&0&1\\1&3&0\end{matrix}\right]$ $\left[\begin{matrix}3&1\\0&3\\1&0\end{matrix}\right]$ = $\left[\begin{matrix}10&3\\3&10\end{matrix}\right]$

Getting eigenvalues from characteristic equation of ATA

Det (λ I-A)=0

$\left|\begin{matrix}λ-10&-3\\-3&λ-10\end{matrix}\right|$ = 0

$(λ-10)^{2}$ – 9 = 0

$λ$2 -20$ λ$ + 100 -9 = 0

$λ$2 -20$ λ$ + 91 = 0

($λ$ -13)($ λ$ -7) = 0

$λ$ = 13 , $λ$ = 7

singular values of the matrix *A :* $σ1$ = $\sqrt{13}$ , $σ2$ = $\sqrt{7}$

**7. Find the maximum value of *z* = 50*x* + 18*y*  subject to
2*x* + *y ≤* 100
*x* + *y ≤* 80
*x ≥* 0*, y ≥* 0
using graphical method**.

For 2x+ y = 100

|  |  |  |
| --- | --- | --- |
| x | 0 | 50 |
| y | 100 | 0 |

2\*0+0$ \leq 100$ True

For x+ y = 80

|  |  |  |
| --- | --- | --- |
| x | 0 | 80 |
| y | 80 | 0 |

0+0$ \leq 100$ True

By solving Two equation above we will get x and y of the point of the intersection of two lines

2x+ y = 100 2x+ y = 100

x+ y = 80 \*-1 -x-y = -80 x = 20 , y = 60

|  |  |
| --- | --- |
| Extreme Point | Value of  *z* = 50*x* + 18*y*   |
| (0,80)(20,60)(50,0)(0,0) | 1440208025000 |

The maximum value is $Z=2500$ and occurs at x = 50 and y = 0