Due Date: 22 October 2015

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Determine whether the statement is true or false:

1. Vectors $\left(7,0,-2\right), \left(4,9,14\right)$ are orthogonal to each other. True
2. $R^{2}is a subspace in R^{3}$. False
3. All linearly independent set in a subspace $W$ is a basis for $W.$ False
4. The transformation$ T:R^{2}\rightarrow R^{2}$, $T(x,y)=2x+3y$ is a linear transformation. True
5. The column space of a $5 × 7$ matrix is in $ R^{5}.$ True
6. If A is $m×n$ matrix then row space of A and column space of A have different dimension. False

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For Each Question, Choose the Correct Answer from the Multiple-Choice List.

1. If $u=(5,1,4)$ and $v=(-1, 0,2) $are two vectors in $R^{3}$. Then the cross product $u×v$:

a. $(-5, 0,8)$

b. $(4,2,6)$

c. $(2,-14, 1)$

d. $(0,0,0)$

1. Let $T\_{1}\left(v\_{1},v\_{2}\right)=\left(v\_{1}-v\_{2},v\_{1}+v\_{2}\right)$ and $T\_{2}\left(v\_{1},v\_{2}\right)=\left(2v\_{2},2v\_{1}\right)$*.* The value of $T\_{1}(T\_{2}\left(v\_{1},v\_{2}\right))$ is:
2. $(2v\_{2}+2v\_{1},2v\_{1}-2v\_{2})$
3. $\left(2v\_{2}-2v\_{1},2v\_{1}+2v\_{2}\right)$
4. $\left(2v\_{1}-2v\_{2},2v\_{1}+2v\_{2}\right)$
5. $\left(2v\_{1}+2v\_{1},2v\_{1}-2v\_{2}\right)$
6. Let $S=\{v\_{1},v\_{2},v\_{3}\}$ is a basis of $V$ and $v\_{2}=3v\_{1}-5v\_{2}.$ Then the coordinate vector of $V$ relative to $S ((v)\_{s})$ is:
7. $(3,5,0)$
8. $(3,0,-5)$
9. $(5,-3)$
10. $(3,-5)$

$4. $A linear combination formed by the vectors$ w\_{1}=\left(1,1,0\right)$, $w\_{2}=\left(0,1,-2\right)$ and $w\_{3}=\left(2,0,4\right)$ is:

1. $w\_{3}=4w\_{1}-3w\_{2}$
2. $w\_{2}=w\_{1}+w\_{3}$
3. $w\_{3}=2w\_{1}-2w\_{2}$
4. $w\_{1}=-w\_{2}-w\_{3}$

Solve the following questions:

1. If $u=(6,-2, -3)$ and $v=(1,1,1)$. Find $cosθ$, where$ θ$ is the angle between $u$ and $v$.

$cosθ$= $\frac{u.v}{\left‖u\right‖\left‖v\right‖}$ = $\frac{\left(6,-12,-3\right).(1,1,1)}{\sqrt{(}6)^{2}+(-2)^{2}+(-3)^{2}. \sqrt{(1)^{2}+(1)^{2}+(1)^{2}}}$

=$\frac{(6)+\left(-2\right)+(-3)}{\sqrt{49} . \sqrt{3}} cosθ$ = $\frac{1}{7\sqrt{3}}$

Let $W=\left\{ a, b ϵ R\right\} $be a subset $R^{3}$. Find a $\left[\begin{matrix}a+2b\\5b\\7a+b\end{matrix}\right]=$$\left[\begin{matrix}a\\0\\7a\end{matrix}\right]+\left[\begin{matrix}2b\\5b\\b\end{matrix}\right]=a\left[\begin{matrix}1\\0\\7\end{matrix}\right] +b\left[\begin{matrix}2\\5\\1\end{matrix}\right].$

$W=\left\{a,b ϵ R\right\} $is a subspace of $R^{3}.$

Let $S=\left\{\left[\begin{matrix}1\\0\\7\end{matrix}\right] ,\left[\begin{matrix}2\\5\\1\end{matrix}\right]\right\}. $

Since $W=Span (S)$, and S is linearly independent.

Then $S$ is a basis for $W$.

1. Let $T\left(u\_{1},u\_{2}\right)=(2u\_{1}, 2u\_{2},2u\_{1}+2u\_{2}$), find the domain, codomain and the image of $\left(1,3\right)$.

Solution:

The domain is $R^{2}$ and the codomain is $R^{3}$.

 The image of $\left(1,3\right)=(2,6,8)$.

1. Given the $3×4$ matrix. Find the basis for Row space of A and its dimension.

$A$=$\left[\begin{matrix}1&4&\begin{matrix}5&2\end{matrix}\\2&1&\begin{matrix}3& 0\end{matrix}\\-1&3&\begin{matrix}2&2\end{matrix}\end{matrix}\right]$

Reducing A to row echelon form



 $R$=$\left[\begin{matrix}1&4&\begin{matrix}5&2\end{matrix}\\0&1&\begin{matrix}1& \frac{4}{7}\end{matrix}\\0&0&\begin{matrix}0&0\end{matrix}\end{matrix}\right]$

the nonzero row vectors of R form a basis for the row space of R and hence form a
basis for the row space of A. These basis vectors are:

$$r1=\left[1 4 5 2 \right]$$

$$r2=[0 1 1 \frac{4}{7} ]$$

and its dimension is 2 (the number of rwos with leading 1 ′s)