Ministry of Higher Education
Kingdom of Saudi Arabia

CSTS
SEU, KSA

Linear Algebra (Math 251)
Level IV, Assignment 1
2016-17

1. State whether the following statements are true or false: [6]

   (a) Every system of linear equation is consistent.

   (a) False

   (b) The addition of two matrices is not possible only when there order differs.

   (b) True

   (c) The transpose of a lower triangular matrix is again lower triangular matrix.

   (c) False

   (d) If $A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$, then $AB = \begin{bmatrix} 6 & 0 \\ 0 & -4 \end{bmatrix}$

   (d) True

   (e) The determinant of every non-singular matrix is zero.

   (e) False

   (f) The absolute values of minors and cofactors of the elements of a square matrix are not identical.

   (f) False

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2. Select one of the alternatives from the following questions as your answer. [6]

   (a) The matrix equation $AX = B$, where $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ corresponds to the system of linear equation

   A.
   $$\begin{aligned} 2x + 3y &= 0 \\ -x - 2y &= 1 \end{aligned}$$

   B.
   $$\begin{aligned} 2x + y &= 0 \\ 3x - 2y &= 1 \end{aligned}$$

   C.
   $$\begin{aligned} 2x - 2y &= 0 \\ 3x - y &= 1 \end{aligned}$$

   D.
   $$\begin{aligned} 2x - y &= 0 \\ 3x - 2y &= 1 \end{aligned}$$

   (b) If If A, B and C are matrices of orders 3×4, 4×5 and 5×2 respectively; then the order of the matrix $(A.B).C$ is

   A. 3×5

   B. 3×4

   C. 3×2

   D. product is not possible.

   (c) If $A = \begin{bmatrix} 2 & 2 \\ 5 & 6 \end{bmatrix}$, then A^{-1} is

   A. $\frac{1}{2}\begin{bmatrix} 6 & -5 \\ -2 & 2 \end{bmatrix}$

   B. $\frac{1}{2}\begin{bmatrix} 6 & 2 \\ -5 & -2 \end{bmatrix}$

   C. $\begin{bmatrix} 3 & -1 \\ -\frac{5}{2} & 1 \end{bmatrix}$

   D. inverse does not exists.

(d) The inverse of a upper triangular matrix is

A. upper triangular

B. lower triangular

C. does not exists

D. any matrix

(e) If $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 2 \\ 3 & -1 & -1 \end{bmatrix}$ then the value of cofactor corresponding to the entry a_{32} is

A. -2

B. 2

C. 14

D. -14

(f) If A is a square matrix of order 3 with $\det(A) = 4$, then $\det(2A)$ is

A. 32

B. 16

C. 8

D. 4

3. Solve the following system of linear equations by Gaussian-elimination method [4]

$$\begin{aligned} x + 2y - 3z &= 1 \\ 2x + 5y - 8z &= 4 \\ 3x + 8y - 13z &= 7 \end{aligned}$$

Solution: The augmented matrix for the given system is

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 2 & 5 & -8 & 4 \\ 3 & 8 & -13 & 7 \end{bmatrix}$$

Adding -2 times the first row to the second and -3 times the first row to the third gives

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 2 & -4 & 4 \end{bmatrix}$$

Adding -2 times second row to third row gives

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The corresponding system of equation is

$$\begin{aligned} x + 2y - 3z &= 1 \\ y - 2z &= 2 \end{aligned}$$

Here we have more equations than unknown variables, so we have infinite number of solutions. Here z is free variable, so choose $z = 1$ and use back substitution, we get

$$x = -4, \quad y = 4, \quad z = 1.$$

4. If $A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 0 & 1 \\ 3 & 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -2 \\ 3 & 8 & -13 \end{bmatrix}$, then find [2]

(a) $A+B$ and $A-B$.

(b) trace of A and B.

Solution:

$(a)\quad A+B=\begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & -1 \\ 6 & 9 & -6 \end{bmatrix}. \quad A-B=\begin{bmatrix} 1 & 1 & 7 \\ -1 & -1 & 3 \\ 0 & -7 & 20 \end{bmatrix}.$

(b) We know that Trace of a square matrix is sum of main diagonal elements. Therefore Trace of the matrix A=9 and trace of the matrix $B=-11$.

5. Verify the property of matrices $(AB)^T=B^TA^T$ for $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $B=\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$. [3]

Solution: By the definition of transpose of a matrix, we have

$$A^T=\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}, \quad B^T=\begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$$
$$B^TA^T=\begin{bmatrix} 24 & 42 \\ 8 & 14 \end{bmatrix}$$
$$AB=\begin{bmatrix} 24 & 8 \\ 42 & 14 \end{bmatrix}$$
$$(AB)^T=\begin{bmatrix} 24 & 42 \\ 8 & 14 \end{bmatrix}$$
$$\therefore \quad (AB)^T=B^TA^T$$

6. Suppose matrix $A=\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then find x and A. [3]

Solution: Since matrix A is symmetric, so $A=A^T$, therefore

$$\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}=\begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$$

Since two matrices are equal only when there corresponding components are equal, so we have $x+2=2x-3$ which yields $x=5$. Therefore matrix A will be $A=\begin{bmatrix} 4 & 7 \\ 7 & 6 \end{bmatrix}$.

7. Find the determinant of the matrix $A=\begin{bmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{bmatrix}$ by cofactor expansion along the second column of A. [3]

Solution: Since cofactor expansion along the second column of A is given by

$$\begin{aligned}|A| &= a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}\\ &= 7(-26) + 1(-26) + 8(26)\\ &= 0\end{aligned}$$

8. Solve the following system of linear equations by Cramer's rule [3]

$$\begin{aligned}4x - 3y &= 15\\ 2x + 5y &= 1\end{aligned}$$

Solution: We know that If $AX = B$ is a system of n linear equations in n unknowns such that $|A| \neq 0$, then the system has a unique solution. This solution is

$$x_1 = \frac{|A_1|}{|A|}, x_2 = \frac{|A_2|}{|A|}, \ldots, x_n = \frac{|A_n|}{|A|}$$

where A_j is the matrix obtained by replacing the entries in the j^{th} column of A by the entries in the matrix B.

Therefore, here

$$\begin{aligned}A &= \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}, \quad So, |A| = 26\\ A_1 &= \begin{bmatrix} 15 & -3 \\ 1 & 5 \end{bmatrix}, \quad So, |A_1| = 78\\ A_2 &= \begin{bmatrix} 4 & 15 \\ 2 & 1 \end{bmatrix}, \quad So, |A_2| = -26\end{aligned}$$

By Cramer's rule, we have

$$x = \frac{|A_1|}{|A|}, \quad y = \frac{|A_2|}{|A|}$$

Therefore, $x = 3, \quad y = -1$.