Math-251 : Linear Algebra Week1

Book: Elementary linear Algebra (Tenth Edition)
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chapter 1: Systems of linear Equations and Matrices

Lecture 1 (Week 1)

(1) 1.1 Introduction to system of linear Equations
(2) 1.2 Gaussian Elimination
(3) 1.3 Matrices and Matrix operation

1.1 Introduction

In 2 dimension a line can be represented by

$ax + by = c$ where $a, b \neq 0$

In 3D a planee can be represented by an equation

$ax + by + cz = d$ where $a, b, c \neq 0$

There are 3 ~~AG~~ ways to ~~sh~~ solve The system of linear equation

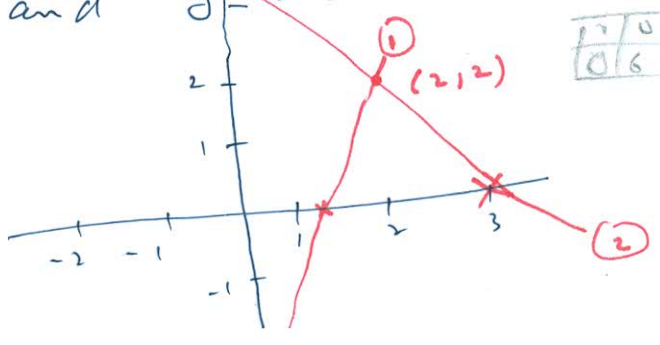
(A) Graphically (B) Algebraically using add or sub
(c) Algebraically using substitution.

(A) Graphically Let we have two lines

$y = 3x - 4$ — (1) and $y = -2x + 6$ — (2)

x	0	4/3
y	-4	0

x	0	3
y	6	0



(3) Now solve (1) & (2) Algebraically

or we can write

$3x - y = 4$ — (1A) $2x + y = 6 \rightarrow$ (2A)

Solve (1A) & (2A)

$$\begin{aligned} 3x - y &= 4 \\ 2x + y &= 6 \\ \hline \cancel{x} &= \cancel{-2} \\ \cancel{x} &= \cancel{-2} \\ 5x &= 10 \\ x &= \tfrac{10}{5} = 2 \end{aligned}$$

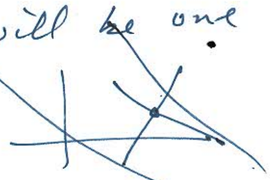
put in (1A) $3 \times 2 - y = 4$

$6 - y = 4$

$6 - 4 = y$

$2 = y$

~~Note: (1) If the lines cross once, there will be one solution~~



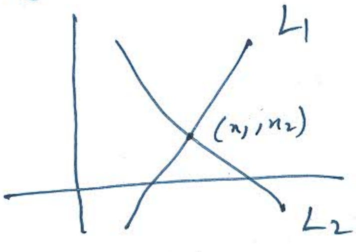
~~(2) If the lines are parallel, There will be no solution.~~

~~(3) I~~

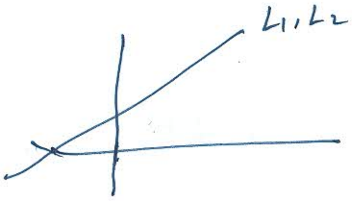
Systems of Equations

If there are two straight lines L_1 and L_2 Then

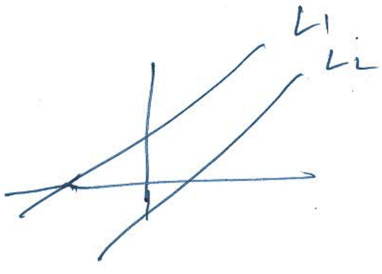
1- If L_1 and L_2 Intersect at exactly one point
Then there is a unique solution



2- If L_1 and L_2 are coincident
Then there are infinitely many solutions



3- If L_1 and L_2 are Parallel
Then there is No solution



Solve the linear equation

$$x - y = 1 \quad -(1)$$
$$2x + y = 6 \quad -(2)$$

Sol: add (1) & (2)

$$\begin{array}{rcl} x - \cancel{y} &=& 1 \\ 2x + \cancel{y} &=& 6 \\ \hline 3x &=& 7 \\ x &=& 7/3 \end{array}$$

put in (1) $\frac{7}{3} - y = 1 \Rightarrow y = \frac{7}{3} - 1$
$= \frac{4}{3}$ //

There is a unique solution $\left(\frac{7}{3}, \frac{4}{3}\right)$

Ex 2: Solve the linear system

$$x + y = 4 \quad -(1)$$
$$3x + 3y = 6 \quad -(2)$$

Solve: we can write

$\times$ by 3, eqn (1)

$$\begin{array}{rcl} \cancel{3x} + \cancel{3y} &=& \cancel{4}\ 12 \\ \cancel{3}x + \cancel{3}y &=& 6 \\ - \quad - & & - \\ \hline 0 - 0 &=& \cancel{12}\ 6 \end{array}$$

$$0 = -6$$

So the given system has no solution.

Ex 3: Solve $4x - 2y = 1 \quad -(1) \quad \times$ by 4
$16x - 8y = 4 \quad -(2)$

Soln:

$$\begin{array}{c} \cancel{16x} - \cancel{8y} = \cancel{4} \\ \underline{16x \mp 8y = 4} \\ 0 = 0 \end{array}$$

This means there are infinitly many solutions in this case we choose any equation and express one varcable in the form of others. for ex.

~~Eg~~ Let eqn (1) be

$4x - 2y = 1$ or $4x = 1 + 2y$ or $x = \frac{1}{4} + \frac{1}{2}y$ — (3)

Now assig any arbitrary value to y, $y = t$ (parameter) Now

$x = \frac{1}{4} + \frac{1}{2}t$ — (4) Now we put any value of t, we got different value of x, Let suppose we put $t = 0$ in (4) Then $x = \frac{1}{4}$ so $(\frac{1}{4}, 0)$ is the Solution

Let $t = 1$ in (4) Then $x = \frac{3}{4}$ So $(\frac{3}{4}, 1)$ is another solution and so on

1.2 Gaussian Elimination

Q1 Use Gauss Elimination to solve the system of linear eqn

$$x_1 + 5x_2 = 7 \quad — (1)$$
$$-2x_1 - 7x_2 = -5 \quad — (2)$$

Soln: Write the Augmented matrix of eqn (1) & (2)

$$\left[\begin{array}{cc:c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right]$$

Now using the Elementary Row operations convert it into Row Echelon form

$$\left[\begin{array}{cc:c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right] \quad R_2 \to R_2 + 2R_1$$

$$\left[\begin{array}{cc:c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right] \quad R_2 \to \frac{R_2}{3}$$

$$\left[\begin{array}{cc:c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 - 5R_2$$

$$\left[\begin{array}{cc:c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right]$$

This is a Row Echelon form Now the value of

$x_1 = -8$ and $x_2 = 3$

Q2 Solve the system of linear equations

$$x - 3y + z = 4$$
$$2x - 8y + 8z = -2$$
$$-6x + 3y - 15z = 9$$

Soln:

The Augmented matrix is

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + 6R_1 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & -5 & -3 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ 0 & -15 & -9 & 33 \end{array}\right] \quad R_2 \to \frac{1}{2} R_2$$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & -15 & -9 & 33 \end{array}\right] \quad R_3 \to R_3 + 15R_2$$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 4 \\ 0 & +1 & -3 & +5 \\ 0 & 0 & -54 & 108 \end{array}\right] \quad R_3 \to \frac{R_3}{-54}$$

$$\left[\begin{array}{ccc:c} 1 & -3 & 1 & 4 \\ 0 & +1 & -3 & +5 \\ 0 & 0 & +1 & -2 \end{array}\right]$$

Now using the Back substitution we can find the value of x, y, z

here $z = -2$

$y - 3z = 5$

$y + 6 = 5 \quad y = 5 - 6$

$y = -1$

$-y + 6 = -5$

$-y = -5 + 6$

$-y = 1$

$y = -1$

$x - 3y + z =$

$x + 3 + (-$

$x + 1 = 4$

$x = 3$

rue false

) If a matrix is reduced row echelon form Then it is also in row echelon form (True)

(2) All leading 1's in a matrix in row echelon form must occure in different columns. (True)

(3) If A and B are 2×2 matrices, Then $AB = BA$ (F)

(4) Trace of the matrix is the product of the elements on the main diagonal (F) (The sum of)

(5) If a matrix is upper triangular or Lower triangular Simultaneously iff it is a diagonal matrix (T)

Q1 Find $A - B^T + \frac{1}{2}C$ if $A = \begin{bmatrix} 7 & 12 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 10 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 14 & 6 \\ 8 & 2 \end{bmatrix}$ and find the trace

Soln:

$$A - B^T + \frac{1}{2}C = \begin{bmatrix} 7 & 12 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 10 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 3 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 5 \\ 4 & 2 \end{bmatrix}$$

The Trace of $A - B^T + \frac{1}{2}C$ is $= 10 + 2 = 12$