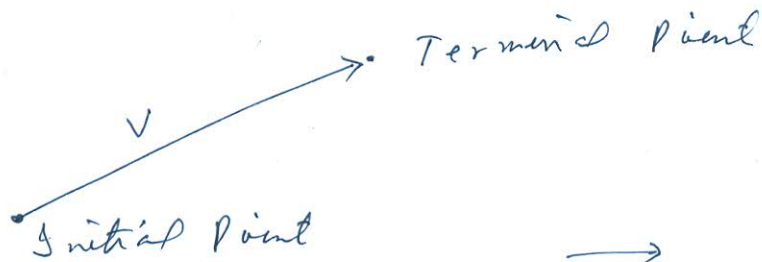


Week - 4, Chapter 3

Vectors

Vectors can be represented in 2-D or 3-D

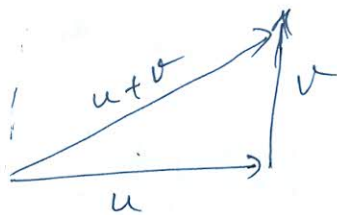


$$v = \overrightarrow{AB}$$

The zero vector can be denoted by 0 , in Two space

$$0 = (0, 0) \text{ in 2-space } 0 = (0, 0, 0)$$

Addition



Components of vector: If we have 2-vectors $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then components of vector are given by

$$\overrightarrow{P_1 P_2} = (x_2 - x_1, y_2 - y_1)$$

Q₁ Find the components of vector with initial point $P_1(2, -1, 4)$ and terminal point $P_2(7, 5, -8)$ are

$$\begin{aligned}\overrightarrow{P_1 P_2} &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (7 - 2, 5 - (-1), (-8) - 4) \\ &= (5, 6, -12)\end{aligned}$$

Q₂ If $v = (1, -3, 2)$ and $w = (4, 2, 1)$, find $v+w$ and $v-w$

Sol: $v+w = (5, -1, 3)$ and $v-w = (-4, -2, 1)$

$$v-w = (-3, -5, 1)$$

Theorem 3.1.1 If u, v and w are vectors in \mathbb{R}^n , and if k and m are scalars, then

(a) $u+v = v+u$ (b) $(u+v)+w = u+(v+w)$

(c) $u+0 = 0+u = u$ (d) $k(u+v) = ku + kv$

(e) $k(mu) = (km)u$ (f) $1 \cdot u = u$

(g) $0 \cdot v = 0$ (h) $k \cdot 0 = 0$

(i) $(-1)u = -u$

Norm of a vector: The Length of a vector v is denoted by $\|v\|$ which is called norm of v or the length of v , or the magnitude of v . So for a vector (u_1, u_2) in

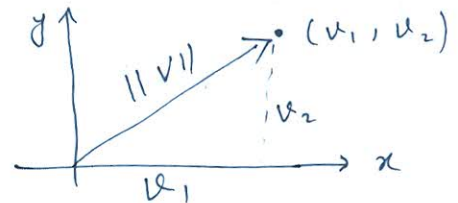
\mathbb{R}^2 $\|v\| = \sqrt{u_1^2 + u_2^2}$

Similarly for a vector (u_1, u_2, u_3) in \mathbb{R}^3

$$\|v\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

Q1 Find the norm of a vector $v = (-3, 2, 1)$ in \mathbb{R}^3 .

Sol: $\|v\| = \sqrt{(-3)^2 + 2^2 + 1^2} = \sqrt{14}$



Unit vector

$$u = \frac{1}{\|v\|} v$$

(2)

Standard or unit vectors in \mathbb{R}^2 or \mathbb{R}^3

$$\text{in } \mathbb{R}^2 \quad \hat{i} = (1, 0) \quad \text{and} \quad \hat{j} = (0, 1)$$

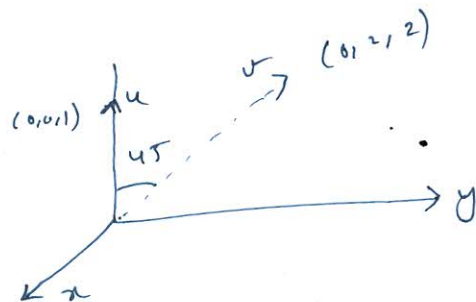
$$\text{in } \mathbb{R}^3 \quad \hat{i} = (1, 0, 0), \quad \hat{j} = (0, 1, 0), \quad \hat{k} = (0, 0, 1)$$

Dot Product: If θ is the angle b/w \vec{u} and \vec{v} , then

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$\text{or} \quad \cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

Q Find the dot product of figure



Soln: $\|u\| = \sqrt{0^2 + 0^2 + 1^2} = 1$

$$\|v\| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2} = 2/\sqrt{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

Now $u \cdot v = \|u\| \|v\| \cos \theta$

$$= (1) (2/\sqrt{2}) (1/\sqrt{2})$$

$$= 2$$

Q Find the dot product of $(u - 2v) \cdot (3u + 4v)$

Soln: $(u - 2v) \cdot (3u + 4v) = u \cdot (3u + 4v) - 2v \cdot (3u + 4v)$

$$= 3(u \cdot u) + 4(u \cdot v) - 6(v \cdot u) - 8(v \cdot v)$$

$$= 3\|u\|^2 - 2(u \cdot v) - 8\|v\|^2$$

~ calculate the dot product of $a = (1, 2, 3)$ and $b = (4, -5, 6)$

Soln: $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$= 1(4) + 2(-5) + 3(6) = 4 - 10 + 18 = 12$$

Q3. If $a = (6, -1, 3)$ for what value of c is the vector $b = (4, c, -2)$ perpendicular to a ?

Sol.

$$a \cdot b = 6(4) - 1(c) + 3(-2)$$

$$= 24 - c - 6$$

$$a \cdot b = 18 - c$$

Now given $a \perp b$ it means $\cos 90 = 0$ so

$$18 - c = 0$$

$$18 = c$$

Orthogonality: Two vectors are orthogonal

$$\text{iff } u \cdot v = 0$$

Q4. Show that $u = (-2, 3, 1, 4)$ and $v = (1, 2, 0, -1)$ are orthogonal in \mathbb{R}^4

Sol.

$$u \cdot v = (-2)(1) + (3)(2) + (1)(0) + 4(-1)$$
$$= -2 + 6 - 4$$
$$= 0$$

\therefore vectors are orthogonal

Theorem 3.3.1 If a and b are constants and non-zero then equation of the form $ax + by + c = 0$ represent a line in \mathbb{R}^2 with normal v

Theorem 3.3.4: In \mathbb{R}^2 the distance b/w the point $P_0(x_0, y_0)$

and the line $ax + by + c = 0$ is $D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

Similarly in \mathbb{R}^3

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Q1. Find the distance b/w the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$ (3)

Soln: or we can write $2x - 3y + 6z + 1 = 0$

$$\text{we know } D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|2(1) + (-3)(-4) + 6(-3) + 1|}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{|-3|}{7}$$

$$= \frac{3}{7}$$

Q2. The two planes are given $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$ are // - Find the distance b/w them.

Soln: we are given $x + 2y - 2z - 3 = 0$ — (1)

$$\text{and } 2x + 4y - 4z - 7 = 0 \text{ — (2)}$$

Let us choose any point on (1) by putting $x=y=0$ then we get

$$x = 3 \text{ so } \text{this is the point on the plane (1) is } (3, 0, 0)$$

Now we know the formula of Dist.

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|2(3) + 4(0) + (-4)(0) - 7|}{\sqrt{2^2 + 4^2 + (-4)^2}} = \frac{1}{6}$$

Defn: If x_0 and \vec{v} are vectors in \mathbb{R}^n , and if v is non-zero,

Then the equation $x = x_0 + tv$ defines a line through x_0 that is

// to v . If $x_0 = 0$ Then the line pass through the origin.

Similarly $x = x_0 + tu_1 + tv_2$ for \mathbb{R}^3

