Week 2, chapter 1

Matrix Properties

(a) $A + B = B + A$ commutative law for addition

(b) $A + (B + C) = (A + B) + C$ (Associative law for add

(c) $A(BC) = (AB)C$ Associative Law for Multip

Note: In Matrix $AB \neq BA$

for ex: Consider $A = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

Then $AB = \begin{bmatrix} -1 & -2 \\ 11 & 4 \end{bmatrix}$ and $BA = \begin{bmatrix} 3 & 6 \\ -3 & 0 \end{bmatrix}$

Thus $AB \neq BA$

Zero Matrix is denoted by 0 for ex:

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$ $\quad \underset{1\times 3}{\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}}$ etc $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$A + 0 = A$$

$$A + (-A) = 0$$

$$0(A) = 0$$

Identity matrix an $n \times n$ matrix with ones on the main diagonal and zeros elsewhere for eg:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Invertible (non singular) : If $AB = BA = I$, Then A is invertable for ex:

$$A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Thus A and B are invertable and each is an inverse of the other.

Theorem 1.4.5 Inverse (old method)

If the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible iff $ad - bc \neq 0$ Then the formula is

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

for ex: (a) $A = \begin{bmatrix} 6 & 1 \\ 5 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$

(a) The determinant $\det(A) = (6)(2) - (1)(5) = 7$

which is non-zero, This A is non-singular or invertable or we can solve it

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{5}{7} & \frac{6}{7} \end{bmatrix}$$

(b) $\det(A) = (-1)(-6) - (2)(3) = 0$

we can't solve it.