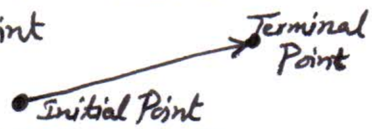
SEC 3.1 VECTORS IN 2-SPACE, 3-SPACE, and n-SPACE

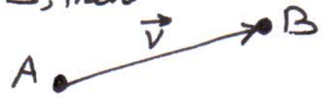
In this Section, we will introduce some of the basic ideas about vectors. As we progress through the text, we will see that Vectors and Matrices are closely related and that much of Linear Algebra is concerned with that relationship.

GEOMETRIC VECTORS: Engineers and physicists represent vectors in two dimensions (also called 2-space) or in three dimensions (also called 3-space) by arrows. The direction of the arrowhead specifies the Direction of the vector and the Length of the arrow specifies the magnitude of the vector. Mathematicians call these Geometric Vectors. The tail of the arrow is called the Initial Point of the Vector and the tip is called Terminal Point.



We will denote vectors in boldface type such as $\vec{a}, \vec{b}, \vec{v}, \vec{w}$ and $\vec{x}$ and we will denote scalars in lowercase italic type such as a, k, v, w, and x.

If a vector $\vec{v}$ has initial point A and terminal point B, then we will write $\vec{v} = \overrightarrow{AB}$



Vectors with the same length and direction are said to be Equivalent. Equivalent Vectors are regarded to be the same vector even though they may be in different positions (See fig.)

Fig. Equivalent Vectors.

The vector whose initial and terminal points coincide has length zero, so we call this the Zero Vector and denote it by $\vec{0}$. The zero vector has no natural direction, so we will agree that it can be assigned any direction that is convenient for problem.

VECTOR ADDITION: There are a no. of important algebraic operations on vectors, all of which have their origin in laws of physics.

Parallelogram Rule for Vector Addition

If $\vec{v}$ and $\vec{w}$ are vectors in 2-space or 3-space that are positioned so their initial points coincide, then the two vectors form adjacent sides of a parallelogram and the sum $\vec{v}+\vec{w}$ is the vector represented by the arrow from the common initial point of $\vec{v}$ and $\vec{w}$ to the opposite vertex of the parallelogram (Fig (i))

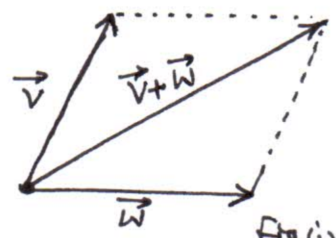


Fig (i)

Here is another way to form the sum of two vectors.

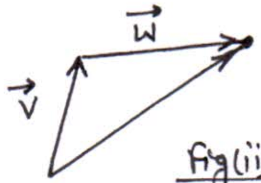


Fig (ii)

Triangle Rule for Vector Addition –

If $\vec{v}$ and $\vec{w}$ are vectors in 2-space or 3-space that are positioned so the initial point of $\vec{w}$ is at the terminal point of $\vec{v}$, then the sum $\vec{v}+\vec{w}$ is represented by the arrow from the initial point of $\vec{v}$ to the terminal point of $\vec{w}$. (See fig (ii))

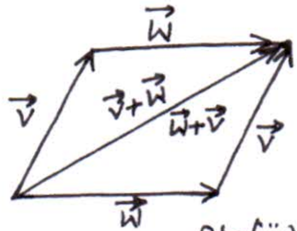


Fig (iii)

NOTE: In fig.(iii), we have constructed the sums $\vec{v}+\vec{w}$ and $\vec{w}+\vec{v}$ by the Triangle rule. This construction makes it evident that

$$\vec{v}+\vec{w} = \vec{w}+\vec{v} \qquad \text{—— (1)}$$

and that the sum obtained by Triangle Rule is same as the sum obtained by Parallelogram rule.

NOTE: Vector Addition $\vec{v}+\vec{w}$ can be viewed as the Translation of $\vec{v}$ by $\vec{w}$ or, alternatively, the Translation of $\vec{w}$ by $\vec{v}$.

VECTOR SUBTRACTION: The Negative of a vector $\vec{v}$ is the vector that has same length as $\vec{v}$ but is oppositely directed and is denoted by $-\vec{v}$. (See fig (i))

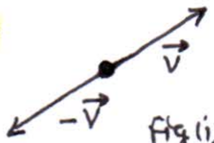


Fig (i)

The Difference of $\vec{v}$ from $\vec{w}$ is taken to be the sum

$$\vec{w}-\vec{v} = \vec{w}+(-\vec{v}) \qquad \text{—— (2)}$$

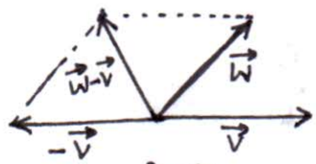


Fig (ii)

The Difference of $\vec{v}$ from $\vec{w}$ can be obtained geometrically by the parallelogram method shown in fig (ii) or more directly by positioning $\vec{w}$ and $\vec{v}$ so their initial points coincide and drawing the vector from the terminal point of $\vec{v}$ to the terminal point of $\vec{w}$ (See fig (iii)).

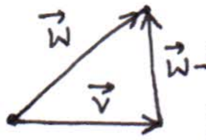
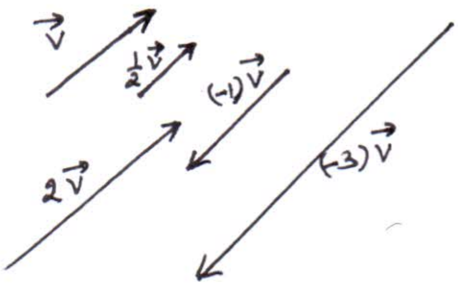


Fig (iii)

SCALAR MULTIPLICATION: If $\vec{v}$ is a non-zero vector in 2-space or 3-space and if k is a non-zero scalar, then we define the Scalar Product of $\vec{v}$ by k to be the vector whose length is $|k|$ times the length of $\vec{v}$ and whose direction is the same as that of $\vec{v}$ if k is +ve and opposite to that if k is −ve. If $k=0$ or $\vec{v}=\vec{0}$, then we define $k\vec{v}$ to be $\vec{0}$

The adjacent fig. shows the geometric relationship between a vector $\vec{v}$ and some of its scalar multiples. In particular, observe that $(-1)\vec{v}$ has same length as $\vec{v}$ but is oppositely directed; therefore,



$$(-1)\vec{v} = -\vec{v} \qquad \text{—— (3)}$$

SUMS OF THREE OR MORE VECTORS.

Vector Addition satisfies the Associative Law for Addition, that is,

$$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$$

A simple way to construct $\vec{u}+\vec{v}+\vec{w}$ is to place the vectors 'tip to tail' in succession and then draw the vector from the initial point of $\vec{u}$ to the terminal point of W (See fig(i)). The 'tip to tail' method also works for four or more vectors (See fig(ii)). The 'tip to tail' method also makes it evident that if $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors in 3-space with a common initial point, then $\vec{u}+\vec{v}+\vec{w}$ is the diagonal of the parallelopiped that has three vectors as adjacent sides (See fig(iii)).

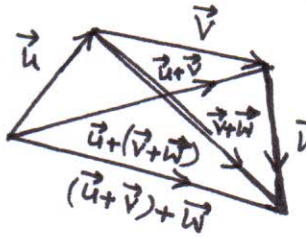


fig (i)

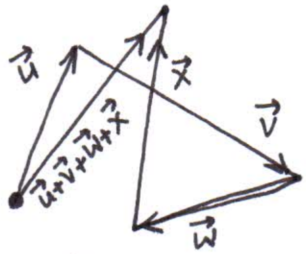


fig (ii)

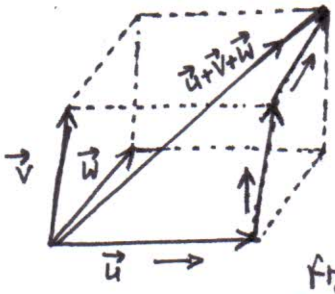
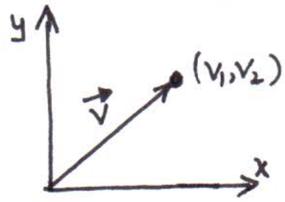
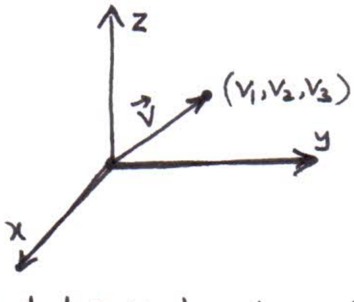


fig (iii)

VECTORS IN CO-ORDINATE SYSTEMS.

If a vector $\vec{v}$ in 2-space or 3-space is positioned with its initial point at the origin of a rectangular co-ordinate system, then the vector is completely determined by the co-ordinates of its terminal point (fig.). We call these co-ordinates the components of $\vec{v}$ relative to the co-ordinate system. We will write $\vec{v} = (v_1, v_2)$ to denote a vector $\vec{v}$ in 2-space with components (v_1, v_2), and $\vec{v} = (v_1, v_2, v_3)$ to denote a vector $\vec{v}$ in 3-space with components (v_1, v_2, v_3).





It should be evident geometrically that two vectors in 2-space or 3-space are Equivalent if and only if they have the same terminal point when their initial points are at the origin. Algebraically, this means that two vectors are Equivalent iff their corresponding components are equal. Thus, for example, the vectors

$\vec{v} = (v_1, v_2, v_3)$ and $\vec{w} = (w_1, w_2, w_3)$ in 3-space are Equivalent iff $v_1 = w_1$, $v_2 = w_2$, $v_3 = w_3$.