## شرح المصفوفة Unitary

**Definition of a Unitary Matrix**

A complex matrix $A$ is **unitary** if

$$A^{-1} = A^*.$$

$$AA^* = A^*A = I$$

**Show that the matrix** $\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$ **is unitary.**

$AA^* = I$

$A^* = \bar{A}^T$

$$\bar{A} = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1+i}{\sqrt{3}} \\ \frac{1-i}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ -\frac{1+i}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$AA^* = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ -\frac{1+i}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(-\frac{1-i}{\sqrt{3}}\right)\left(-\frac{1+i}{\sqrt{3}}\right) & \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1-i}{\sqrt{3}}\right) + \left(-\frac{1+i}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) \\ \left(\frac{1+i}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(-\frac{1+i}{\sqrt{3}}\right) & \left(\frac{1+i}{\sqrt{3}}\right)\left(\frac{1-i}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

is unitary