Saudi Electronic University / College of Computing and Informatics
Assignment of week 2/ Math 150

Q 1: Determine whether of the statement true or false:

1- $x^2 + y^2 = z^2$ is not proposition. False
2- The conditional statement $p \rightarrow q$ is false when both p and q are false. False
3- $\sim(\sim p \vee \sim q)$ and $p \wedge q$ are logically equivalent. True
4-$p \vee \sim p$ Is a contradiction. False
5- The biconditional statement $p \leftrightarrow q$ is false when both p and q are false. False

Q 2: Choose the correct answer:

1- The negation of the statement $\sim p \rightarrow q$ is:
a- $p \wedge q$ b- $p \wedge \sim q$ c-$\sim p \wedge \sim q$ d-$\sim p \wedge q$

2- The inverse of the statement $p \rightarrow \sim q$ is:
a- $\sim p \rightarrow q$ b- $p \rightarrow q$ c- $\sim p \rightarrow \sim q$ d-$p \rightarrow \sim q$

3- $x \oplus y = 1$ When:
a-$x = 0, y = 1$ b- -$x = 0, y = 0$ c- -$x = 1, y = 1$ d-None

4- One of the following is proposition:
a- When will go to school? B - x + y=1 c- x-y=5 d-Riyadh is the capital of Saudi Arabia

5- One of the following is a tautology:
a-$p \wedge p$ b-$p \vee p$ c-$p \wedge \sim p$ d-$p \vee \sim p$

Q 3:

1- **Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent**

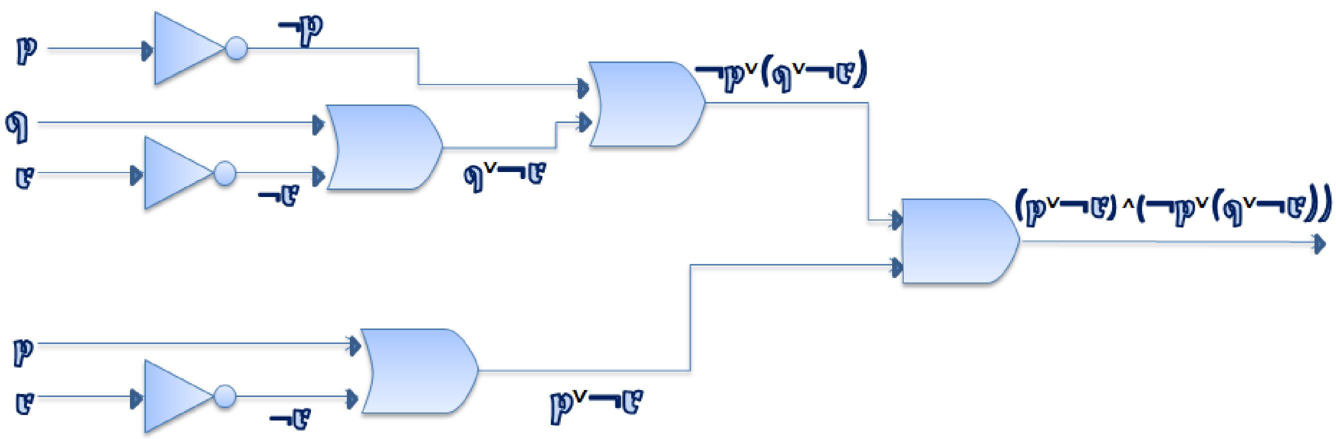
$\neg(p \vee (\neg p \wedge q))$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
	$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
	$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$
	$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$	by the commutative law for disjunction
	$\equiv \neg p \wedge \neg q$	by the identity law for F

p	q	¬p	¬q	¬p∧¬q	(p∨(¬p∧q))	¬(p∨(¬p∧q))
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

They are equivalent

2- Build a digital circuit that produces the output

$(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$ when given input bits *p*, *q*, and *r*.



3- Find the bitwise OR of the bit strings 011011001 and 000011110.

Bitwise OR = 011011111

4- Find the bitwise XOR of the bit strings 0011001 and 0000111.

Bitwise XOR = 0011110

5- Use a truth table to verify $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

p	q	¬p	¬q	p∧q	¬(p∧q)	¬p∨¬q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

They are equivalent

Saudi Electronic University / College of Computing and Informatics
Assignment of week 3/ Math 150

Section 1 **Section A: (True or False Questions)**

1- The OR function is Boolean multiplication and the AND function is Boolean addition. **False**

2-In Boolean algebra theorem is very useful for minimizing logic systems **True**

2- In Boolean algebra, A + 1 = 1. **True**

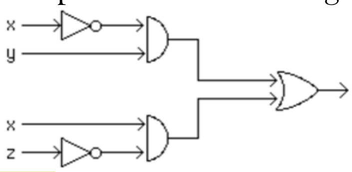
3-the value of $1 \cdot 0 + \overline{(0 + 1)}$ is 0. **True**

4- The domination law is x.1=x. **False**

5- $\bar{x}$ = x | x. **True**

Section 2

1-Derive the Boolean expression for the logic circuit shown below:



a-$\bar{x}y + xz$ b-$\boldsymbol{\bar{x}y + x\bar{z}}$ c-$\boldsymbol{x\bar{y}} + \boldsymbol{xz}$ d- None of above

2- The Boolean expression C + CD is equal to

a- C   b- D   c- C+D   d- 1

3- How many different Boolean functions are there of degree 3?

a-16   b-4   c-256   d- 18

4- (x | y) | (x | y) is equal to

a- x+y   b- xy   c- x(x+y)   d- None of the above

5-(x | x) | (y | y) equal to

a-$\bar{y}x$ b- xy   c-x+y   d- $\bar{x}y$

Section 3

1-Prove the absorption law x(x + y) = x using the other identities of Boolean algebra

$x(x + y) = x$

$xx + xy = x$

$x + xy = x$

$x(1 + y) = x$

$x \cdot 1 = x$

$\therefore So:\ x(x + y) = x$

2-Find the duals of x(y + 0)

$x + (y \cdot 1)$

3-Give a Boolean expression for the Boolean function F(x, y) as defined by the following table:

x	y	F(x, y)
0	0	0
0	1	1
1	0	0
1	1	0

F(x, y) = 1 when x = 0 and y = 1 and 0 otherwise (that could be established by Boolean products or by using NAND operator)

x	y	$\bar{x}$	$\bar{y}$	$\bar{x}y$	$x \vert \bar{y}$
0	0	1	1	0	0
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	0	0

4- Find the values, if any, of the Boolean variable x*s that satisfy* these equations.

$x \cdot \bar{x} = 1$ None

$\mathbf{x \cdot 1 = x}$ 0, 1

5- Find the sum of products expansion of a Boolean function f(x, y, z) that is 1 if and only if x = y = 1 and z = 0 or x = 0 and y = z = 1 or x = y = 0 and z = 1

x = y = 1 and z = 0 ➔ $xy\bar{z}$

Or x = 0 and y = z = 1 ➔ $\bar{x}yz$

Or x = y = 0 and z = 1 ➔ $\bar{x}\bar{y}z$

$\therefore\ so\ (xy\bar{z}) + (\bar{x}yz) + (\bar{x}\bar{y}z)$

x	y	z	F(x, y, z)
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

Answer Key Discrete Mathematics (Math 150) Level III, Week 4 (2014-15)

1. State whether the following statements are true or false:
   a) $\neg \forall x\, P(x) \equiv \exists x\, P(x)$ **F**
   b) $\neg \exists x\, Q(x) \equiv \forall x\, \neg Q(x)$ **T**
   c) $\forall x\, \exists y\, Q(x; y)$ states that for every x there does not exist any y such that Q(x; y) **F**
   d) If n is odd integer, then n^2 is odd. **T**
   e) 9 x P(x) is the universal quantification. **F**

2. Select one of the alternatives from the following questions as your answer.
   a) Universal quantifier is denoted by
      A. $\exists$
      B. $\forall$
      C. $\aleph$
      D. None
   b) Existential quantifier is denoted by
      A. $\exists$
      B. $\forall$
      C. $\aleph$
      D. None
   c) The statement $\neg\exists\, xP(x)$ is equivalent to
      A. $\exists x\, \neg P(x)$
      B. $\exists x\, P(x)$
      C. $\forall x\, P(x)$
      D. $\forall x\, \neg P(x)$
   d) The statement $\neg \forall x(P(x) \rightarrow Q(x))$ is equivalent to
      A. $\exists x\, (\neg P(x) \rightarrow \neg Q(x))$
      B. $\exists x\, (P(x) \vee \neg Q(x))$
      C. $\exists x\, (P(x) \wedge \neg Q(x))$
      D. None
   e) The sum of two positive integer is always positive. Its logical translation is
      A. $\forall x\, \exists y\, ((x > 0) \rightarrow (x + y > 0))$
      B. $\forall x\, \forall y\, ((x > 0) \wedge (y > 0) \rightarrow (x + y > 0))$
      C. $\forall x\, \exists y\, ((x > 0) \wedge (y > 0) \rightarrow (x + y > 0))$
      D. None

3. Show that the sum of two rational numbers is rational.

Direct proof: M and N are rational numbers, $M = \frac{a}{b}; b \neq 0$ and $N = \frac{c}{d}; d \neq 0$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd} = \frac{R}{S}$, so the sum of two rational numbers is a rational number

4. Show that if n is an integer and 3n + 2 is odd, then n is odd.

Direct proof: N = “n is integer and 3n + 2 is odd”, S = “n is odd” and (N →S), supposing that if N than S is true so: $3n + 2 = 2k + 1$ ➔ $n = \frac{2k-1}{3}$ which is not a formula for odd numbers.

Indirect proof (proof by contraposition): supposing that if N than S is false and n is even number, so: $n = 2k$, $3n + 2$ ➔ $3(2k) + 2$ ➔ $6k + 2 = 2(3k + 1) = 2n$ and that proof 3n+2 to be even, that as a negation of S that lead to negation of N to be false contrasting the original statement then the original statement is true.

5. Let P(x) denote the statement “$x \le 4$": What are the truth values of P (0); P (4); P (6):

P (0) = $0 \le 4$, which is true; P (4) = $4 \le 4$, which is true; P (6) = $6 \le 4$, which is false.

6. Let P(x): $\forall x (x^2 > x)$ and Q(x): $\exists x (x^2 = 2)$. Find ¬P(x), ¬Q(x):

$\neg P(x) = \neg \forall x (x^2 > x) = \exists x (x^2 < x)$

$\neg Q(x) = \neg \exists x (x2 = 2) = \forall x (x^2 \neq 2)$

7. Let P(x): “x spends more than five hours every weekday in class", where the domain for x consists of all students. Express each of the following quantifications in English:

a) **$\exists x P(x)$**

There exists student how spends more than five hours every weekday in class.

b) **$\forall x P(x)$**

All students spend more than five hours every weekday in class.

c) **$\exists x \neg P(x)$**

There exists student how doesn’t spend more than five hours every weekday in class

d) **$\forall x \neg P(x)$**

All students don’t spend more than five hours every weekday in class.

Saudi Electronic University
College of Computing and Informatics
Discrete Mathematics (Math 150) Assignment week 5

Section A

Determine whether each of these statements is true or false

1. $\{x\} \subset \{x, \{x\}\}$. T
2. The function $f(x) = \frac{x+1}{x+2}$ is **not** a bijection from **R** – {- 2} to **R** – {1}. F
3. Suppose that A is an $n \times n$ matrix where n is a positive integer. Then $A + A^t$, AA^t is symmetric.
4. The seventh term a_6 of the sequence defined by the recurrence relation and initial conditions, $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1, a_1 = 1, a_2 = 2$ is 1. F
5. Let A , B and C be three square matrices of same size. Then $(A + BC)^t = A^t + B^tC^t$.

Section B

Each of the multiple choice objective questions contain four answers, of which one correct answer is to be marked.

1) Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Then the set $\bar{A} \cup \bar{B}$ is expressed by

a) The set of sophomores taking discrete mathematics in your school.
b) The set of sophomores at your school who are not taking discrete mathematics.
c) The set of students at your school who either are sophomores or are taking discrete mathematics.
d) The set of students at your school who either are not sophomores or are not taking discrete mathematics.

2) What is the truth set of the predicate P(x), where the domain is the set of integers and P(x): $< x^2$.

a) {0, 1} b) $\{x \in \boldsymbol{Z} \mid x \neq 0 \wedge x \neq 1\}$ c) $\{x \in \boldsymbol{Z} \mid x \neq 0 \vee x \neq 1\}$ d) $\boldsymbol{Z}^+$.

3) Determine which of the function $f: \boldsymbol{Z} \times \boldsymbol{Z} \to \boldsymbol{Z}$ is **not** onto

a) $f(m,n) = 2m - n$ b) $f(m,n) = m^2 - n^2$
c) $f(m,n) = m + n + 1$ d) $f(m,n) = |m| - |n|$.

4) Which of the below given sequences $\{a_n\}$ is **not** a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$?

a) $n4^n$ b) 0 c) 2^n d) 4^n

5) Let A be a 3×4 matrix, B be a 4×5 matrix, and C be a 4×4 matrix. Determine which of the following products is **not** defined?

a)AB b) AC c) CA d) CB.

Section C

1) Let $\boldsymbol{f(x) = x^2}$ be the function from R to R. Find

$\boldsymbol{a)}$ $\boldsymbol{f^{-1}(\{1\})}$ b) $\boldsymbol{f^{-1}(\{x|\ x > 4\})}$.

$a)$ $f^{-1}(\{1\}) = x$ ➔ $f(x) = \{1\}$, $x^2 = 1$ ➔ $x = 1$ or $x = -1.$

$\therefore S.S = \{-1,1\}$

$b)$ $f^{-1}(\{x|\ x > 4\}) = y$ ➔ $f(y) = \{x|\ x > 4\}$ ➔ $y^2 = \{x|\ x > 4\}$ ➔ $y^2 > 4$ ➔

$\{y > 2$ or $y < -2\}$.

$\therefore S.S = \{x|\ x > 2 \vee x < -2\}$

2) Use set builder notation and logical equivalences to establish the second De Morgan´s law $\overline{\boldsymbol{A \cup B}} = \overline{\boldsymbol{A}} \cap \overline{\boldsymbol{B}}$.

$\overline{A \cup B} = \{x \mid x \notin A \cup B\}$ ➔ $\{x \mid \neg (x \in (A \cup B))\}$

$\{x \mid \neg (x \in A \vee x \in B)\}$ ➔ $\{x \mid \neg (x \in A) \wedge \neg (x \in B)\}$

$\{x \mid (x \notin A) \wedge (x \notin B)\}$ ➔ $\{x \mid (x \in \overline{A}) \wedge (x \in \overline{B})\}$

$\{x \mid x \in \overline{A} \cap \overline{B}\}$ ➔ $\overline{A} \cap \overline{B}$

3) Find $\boldsymbol{fog}$ and $\boldsymbol{gof}$, where $\boldsymbol{f(x) = x^2 + 1}$ and $\boldsymbol{g(x) = x + 2}$, are functions from R to R.

$(fog)(x) = f(g(x)) = f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 5$

$(gof)(x) = g(f(x)) = g(x^2+1) + 2 = x^2 + 3$

4) Find the value of each of these sums

$\boldsymbol{a)}$ $\boldsymbol{\sum_{j=0}^{8} (1 + (-2)^j)}$ b) $\boldsymbol{\sum_{i=1}^{3} \sum_{j=0}^{2} (j)}$.

a) $\sum_{j=0}^{8} (1 + (-2)^j) = (1 + (-2)^0) + (1 + (-2)^1) + (1 + (-2)^2) + (1 + (-2)^3) + (1 + (-2)^4) + (1 + (-2)^5) + (1 + (-2)^6) + (1 + (-2)^7) + (1 + (-2)^8) = 180$

b) $\sum_{i=1}^{3} . \sum_{j=0}^{2} j = 3 \cdot (0 + 1 + 2) = (0 + 1 + 2) + (0 + 1 + 2) + (0 + 1 + 2) = 9$

5) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find

a) $A \vee B$ b) $A \odot B$.

Saudi Electronic University
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Assignment of week 6/ Math 150

Section A: (True or False Questions)

1. $f(x)=x+x^3$ is $O(x^3)$ T
2. 3644 mod 645 equal 35 F
3. If $f_1(x)$ and $f_2(x)$ are both $O(g(x))$. Then $(f_1+f_2)(x)$ is $O(g(x))$. T
4. An algorithm has input values from a specified set T

Section A: (Choose correct answer)

1. Which of these functions is not $O(x^2)$.
   a) $f(x)=17x+11$ b) $f(x)=x^2+1000$
   c) $f(x)= x\log x$ d) $f(x)=x^4/2$

2. The worst-case complexity of the bubble sort in terms of the number of comparisons
Made is.

a $\frac{n^2-n}{2}$ b $\frac{n^2+n}{2}$ c $\frac{n+1}{2}$ d $\frac{n^3-n}{2}$

3. The procedure double (n: positive integer)
While n>0
n:= 2n

A) Procedure is not finite. B) Procedure is not effective C) procedure lacks definiteness D) Procedure is effective

4 let set 1,3, 4, 5, 6, 8, 9, 11. Then the procedure m ($a_1,a_2,\ldots,a_n$: integers)
m:= a1
For i: = 2 to n
If max > a_i then m:= a_i
Return m

A) m=9 B) m=11 C) m=1 D)m=6

Section 3

1. Describe an algorithm that takes as input a list of n integers and finds the number of negative integers in the list.

"To retrieve only the number of negative integer no it's value"

Procedure negatives (a_1, a_2, … , a_n: integers)

i:=0

for k := 1 to n

if a_k < 0 then i++ <=> (i := i+1)

Return i ;

2. List all the steps used to search for 7 in the sequence 1,3, 4, 5, 6, 8, 9, 11 for both a linear search and a binary search.

- Linear search

1	3	4	5	6	8	9	11
7>1 ↓	7>3 ↓	7>4 ↓	7>5 ↓	7>6 ↓	7<8 ↓	7<9 ↓	7<11 ↓
Next ↗	Next ↗	Next ↗	Next ↗	Next ↗	Next ↗	Next ↗	Not found

Number 7 is not found the output is 0.

- Binary search

1, 3, 4, 5, 6, 8, 9, 11 ➔ split it in two groups

1, 3, 4, 5 6, 8, 9, 11

7>5 so it's not in the first group, 6<7<11 so it's in the second group

6, 8 9, 11

6<7<8 so it's in the first group, 7<9, 11 so it's not in the second group

In the first group it's either 6 or 8 and no sign of 7 so it's not found and the output is 0.

3. Show that $f(x)=x^2+2x+1$ is $O(x^2)$.

$|f(x)| \le C \cdot |g(x)|$ ➔ $when\ x > k \to x > 1, f(x) \le C|g(x)|$

$= x^2 + 2x + 1 \le C(x^2)$ ➔ $= x^2 + 2x + 1 \le (x^2 + 2x^2 + x^2) = 4x^2$

So taken C= 4 and k=1 as witnesses we say that $f(x)\ is\ O\ (x^2)$

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Assignment of week 7/ Math 150

Section 1: (True or False Questions)

1/ 7/3 is a integer. F

2 Let a,b,c be integers where a≠ **0** then . if a/b and a/c , then a/(a+b) . T

3 In the binary notation each digit is either a , 0 or a, 1. T

4 Convert $(101011)_2$ to base 8 = $(50)_8$. F

5 If p is the prime and a is an integer not divisible by p.
Then $a^{p-1} \equiv 1 \pmod p$ T

Section 2: (Multiple choice questions)

1 Let a = 101 and b= 16 what are the values .of q and r that division algorithm . Guarantees in this case ?

a> q =6 , r =5 b> q =5 , r = 6 c> q = 5 , r = 6 d> q = 7 , r = -11

2what are quotient, when 101 is divided by 11 .

a> Hence the quotient when 101 is divided by 11 is 1 = 101 div 11
b> Hence the quotient when 101 is divided by 11 is 0 = 101 div 15
c> Hence the quotient when 101 is divided by 11 is 8 = 101 div 10
d> Hence the quotient when 101 is divided by 11 is 9 = 101 div 11

3 convert $(204)_{10}$ to base 2 .

a> 10001000 b> 11001100 c> 00001111 d> 00000000

4 Find the prime factorization of 1024.

a> 2^{10} b> 3^{10} c> 0 d> 1

5 Find an inverse of 5 modulo 12.

a> 5 b> 0 c> -1 d> 10

Section 3 Five small question

1. what is the decimal expansion of the integer that $(101011111)_2$ as its binary expansion .

$1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 351$

2. Find 7^{222} mod 11 .

   We know that $7^{10} \equiv 1 \pmod{11}$, so $(7^{10})^k \equiv 1 \pmod{11}$, for every positive integer k. Therefore,

   $7^{222} = 7^{10 \cdot 22 + 2} = (7^{10})^{22} \times 7^2 \equiv (1)^{22} \times 49 = (\bmod\ 11) \equiv 5 \pmod{11}$.

   So 7^{222} mod 11 = 5.

3. what is the decimal expansion of the number with hexadecimal expansion $(2AE0B)_{16}$?

$2 \times 16^4 + 10 \times 16^3 + 14 \times 16^2 + 0 \times 16^1 + 11 \times 16^0 = 175627$

4. Use the Euclidean algorithm to find

   (a) gcd(203, 101).

   $203 = 101 \times 2 + 1$

   $101 = 1 \times 101 + 0$

   So gcd (203, 101) = 1

   (b) gcd(34, 21).

   $34 = 21 \times 1 + 13$

   $21 = 13 \times 1 + 8$

   $13 = 8 \times 1 + 5$

   $8 = 5 \times 1 + 3$

   $5 = 3 \times 1 + 2$

   $3 = 2 \times 1 + 1$

   $2 = 1 \times 1 + 0$

   So gcd (203, 101) = 1

5. Find the prime factorization of 111111.

   111111/3 = 37037➔ 37037 /7 = 5291 ➔5291 /11 = 481 ➔481 /13 = 37

   So the prime factorization is $3 \times 7 \times 11 \times 13 \times 37$

Saudi Electronic University / College of Computing and Informatics
Discrete Mathematics (Math 150) / Assignment week 8

Section A Determine whether each of these statements is true or false

1) The first five Fibonacci numbers are: 1,2,3,5,8. F
2) A simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n-2$ triangles. T
3) $f(0)=0, f(n)=2f(n-2)$ for $n \geq 1$ is a recursive definition of a function f. F
4) The recursive definition of the set $S=\{1,5,9,13,17,\ldots\}$ is $1 \in S; x \in S \rightarrow x+4 \in S$. T
5) Mathematical induction, strong induction and well ordering are not equivalent principles. F

Section B Each of the multiple choice objective questions contain four answers, of which one correct answer is to be marked.

1) The sums of the first $\boldsymbol{n}$ positive odd integers are
   a) $2n+2$
   b) n^2
   c) $(n+1)(n-1)$
   d) $(2n-1)n$

2) Let $\boldsymbol{P(n)}$ be the statement $\mathbf{1^3+2^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2}$ for the positive integer n. Then what is the value of $\boldsymbol{P(2n-1)}$

a) $(2n-1)^2 n^2$ b) $\frac{(2n-1)(2n+1)}{2}$ c) $\left(\frac{(2n-1)(2n+1)}{2}\right)^2$ d) none

3) The sum of 1+2+3+...+25 is

a) 320 b) 325 c) 425 d) 300.

4) The recursive definition of the set S of positive integers that are multiples of 5 is

a) $5 \in S$, and $x+y \in S$ if $x,y \in S$ b) $5 \in S$, and $5y \in S$ if $y \in S$
c) $5 \in S$, and $x.y \in S$ if $x,y \in S$ d) $5 \in S$, and $\frac{x}{y} \in S$ if $x,y \in S$

5) The recurrence definition with initial condition of a function $f(x)=5n+2, n=1,2,3\ldots$ is

a) $f(n)=2f(n-1)+5, f(1)=2$ b) $f(n)=f(n-1)+5, f(1)=7$
c) $f(n+1)=f(n)+7, f(1)=4$ d) $f(n)=f(2n+1)+5, f(1)=7$

Section C Short answer type questions

1) Use mathematical induction to show that

$$1 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$

$P(n) = 1 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$

Basis Step:

$P(0)$ is true becouse $2^0 = 2^{0+1} - 1 = 1$

Inductive Step:

- ⇨ Assume $P(k)$ is true for all nonnegative integer k: $1 + 2^1 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$
- ⇨ Must Show $P(k+1)$ is true:
  $1 + 2^1 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{(k+1)+1} - 1$
  $1 + 2^1 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{k+2} - 1$
- ⇨ Assume $P(k)$ is true

$$1 + 2^1 + 2^2 + \cdots + 2^k + 2^{k+1} = (1 + 2^1 + 2^2 + \cdots + 2^k) + 2^{k+1}$$
$$= (2^{k+1} - 1) + 2^{k+1}$$
$$(2^{k+1} - 1) + 2^{k+1} = 2^{k+1} + 2^{k+1} - 1 = 2^{k+1} \cdot 2 - 1$$
$$2^{k+1} \cdot 2^1 - 1 = 2^{(k+1)+1} - 1 = 2^{k+2} - 1$$

$\therefore$ $P(k)$ is true $\rightarrow$ $P(k+1)$ is true

2) Give a recursive algorithm for computing na using addition, where n is a positive integer and a is a real number.

3) Show that if n is an integer greater than 1, then n can be written as the product of primes.

P(n) = n can be written as the product of primes.

Basis Step:

P (2) is true, because 2 can be written as the product of one prime.

Inductive Step:

- ⇨ Assume P(j) is true for all integers j with 2 ≤ j ≤ k.
- ⇨ Must show: P(k + 1) is true, and k + 1 is product of primes.
  - ➔ 1) If k + 1 is prime, then P(k + 1) is true.
  - ➔ 2) If k + 1 is composite and can be written as the product of two positive integers M and N with 2 ≤ M ≤ N < k + 1.

  Because both M and N are integers at least 2 and not exceeding k, we can write both M and N as the product of primes. Thus, if k + 1 is composite, it can be written as the product of primes, those who are factorization of M and N.

4) Find $f(1), f(2), f(3)$ and $f(4)$, if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 0, 1, 2, 3, \ldots$

a) $f(n+1) = f(n) + 2$

$f(0+1) = f(0) + 2 \Rightarrow f(1) = 1 + 2 = 3$

$f(1+1) = f(1) + 2 \Rightarrow f(2) = 3 + 2 = 5$

$f(2+1) = f(2) + 2 \Rightarrow f(3) = 5 + 2 = 7$

$f(3+1) = f(3) + 2 \Rightarrow f(4) = 7 + 2 = 9$

b) $f(n+1) = 2^{f(n)}$.

$f(0+1) = 2^{f(0)} \Rightarrow f(1) = 2^1 = 2$

$f(1+1) = 2^{f(1)} \Rightarrow f(2) = 2^2 = 4$

$f(2+1) = 2^{f(2)} \Rightarrow f(3) = 2^4 = 16$

$f(3+1) = 2^{f(3)} \Rightarrow f(4) = 2^{16} = 65536$

5) Give a recursive definition with initial condition of

a) $P_m(n)$, the product of the integer m and the non-negative integer n.

$P_m(0) = 0\ , P_m(n+1) = P_m(n) + m$

b) The set of positive integers not divisible by 5

$1 \in S,\ 2 \in S,\ 3 \in S,\ 4 \in S$ and if $x \in S$, then $x + 5 \in S$

c) The set $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$.

$0 \in S$, and if $x \in S$, then $x + 2 \in S$ and $x - 2 \in S$

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Discrete Mathematics (Math 150) / Assignment week 11

Q 1: Determine whether each of these statements is true or false:

1-the number of function from a set with 3 elements to a set with 6 elements is 18. F

2- the number of different bit strings of length 5 is 32. T

3-$P(n,0) = 0$. F

4- $\boldsymbol{C(n,r) = \frac{n!}{(n-r)!r!}}$. T

$\mathbf{5 - \sum_{k=0}^{n} (-1)^{K} \binom{n}{k} = -1}$ F

Q 2: Choose the correct answer:

1-The coefficient of $x^{12}y^{12}$ in the expression of $(3x-2y)^{24}$ is :

a) $\binom{24}{12}3^{12}2^{12}$ b) $\binom{24}{12}\mathbf{3^{12}(-3)^{12}}$

b) $3^{12}2^{12}$ d) $\mathbf{3^{12}(-2)^{12}}$

2-The number of different strings can be made by reordering the letters of the word CORPORATION is

$a)\ \frac{11!}{2!3!}$ b) $\frac{11!}{5!}$ c) $\frac{11!}{2!3!6!}$ d) $\frac{11!}{5!6!}$

3-The minimum number of students required in a class to be sure that at least 10 will receive the same grade if there are 6 possible grades is:

a) 60 b) 10^6 c) 6^{10} d) 55

4- $\sum_{k=0}^{10}\binom{10}{k}$ =:

a)10 b) 1 c) 1024 d) 0

5- The number of 1-1 functions from a set with 4 elements to a set with 7 elements is:

a) 840 b) 11 c) 28 d) 3

Q 3: Solve the following questions:

1- Somebody can choose a picture from one of four lists. The four lists contain 18,6,12,4 possible pictures, respectively. No picture is on more than one list. How many possible pictures are there to choose from

18 + 6 + 12 + 4 = 40 picture

2- From a group of 7 men, 3 men are to be selected to form a committee. In how many ways can it be done?

$$C(7,3) = \frac{7!}{(7-3)!\cdot 3!} = \frac{7\times 6\times 5\times 4!}{4!\cdot 6} = 35$$

3- How many 3-digit number can be done from the digits 1, 2,3,4,5, and 6, and repeating is not allowed.

$$P(6,3) = \frac{6!}{(6-3)!} = \frac{6\times 5!}{3!} = 120$$

4- How many 3-digit number can be done from the digits 1, 2,3,4,5, and 6 which is even and repeating is not allowed.

Last number should be even and one of 3 choses (2, 4, 6) so

$5\times 4\times 3 = 60$

Or it's simply half of 120

5- Write the expansion of $(2x-y)^4$.

$$(2x-y)^4 = \sum_{j=0}^{4}\binom{4}{j}$$

$$= (2x)^{4-j}(-y)^j = \binom{4}{0}(2x)^4 + \binom{4}{1}(2x)^{4-1}(-y) + \binom{4}{2}(2x)^{4-2}(-y^2) + \binom{4}{3}(2x)^{4-3}(-y^3) + \binom{4}{4}(2x)^{4-4}(-y^4)$$

$$= 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$$

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Discrete Mathematics (Math 150) / Assignment week 11

Question Number (1):
Determine whether each of these statements is true or false?

1) The recurrence relation $p_{n=(1.11)p_{n-1}}$ is a linear homogeneous recurrence relation of degree one. T

2) The recurrence relation $a_{n=a_{n-1}+a^2{}_{n-2}}$ is linear. F

3) The solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1 \ and \ a_1 = 6$ is given by $a_n = 3^n + n3^n$ T

4) Suppose that $f(n)$ that satisfies the divide-and- conquer recurrence relation $f(n) = 3f\left(\frac{n}{4}\right) + \frac{n^2}{8}$ with $f(1) = 2 \ then \ f(64) = 680$ T

5) The recurrence relation $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$ is a linear homogeneous recurrence relation of degree 3 T

Question Number (2):
Choose the correct answer?

1) Find the solution of the recurrence relation $(a_n = 3a_{n-1})$ with $a_0 = 2$
   a)2.3^n b)2^n c)3^{n+1} d)3^n

2) find the solution of a linear homogeneous recurrence relation $a_n = 7a_{n-1} - 6a_{n-2}$ with $a_0 = -1 \ and \ a_1 = 4$
   a)$a_n = 6^n$ b)$a_n = -2 + 6^n$ c)2^n d)2^{n+1}

3) Suppose that $f(n)$ satisfies the divide-and-conquer relation $f(n) = 2f\left(\frac{n}{3}\right) + 5$ and $f(1) = 7$ what is $f(81)$?
   a) 180 b) 187 c)170 d)185

<u>**Question Number (3):**</u>

Solve the following questions?

1) Solve recurrence relation together with the initial condition given $a_n = 2a_{n-1}$ for ≥ 1 , $a_0 = 3$?

$a_n = 2a_{n-1}$, $r = 2$, $\alpha = a_0 = 3$
$a_n = 3 \cdot 2^n$
When n =1, $a_n = 6$

2) Multiply $(1110)_2$ $and(1010)_2$ using the fast multiplication algorithm?

$(\mathbf{1110})_2(\mathbf{1010})_2 = (\mathbf{2^4 + 2^2})(\mathbf{11})_2(\mathbf{10})_2 + \mathbf{2^2}[(\mathbf{11})_2 - (\mathbf{10})_2][(\mathbf{10})_2 - (\mathbf{10})_2] + (\mathbf{2^2 + 1})(\mathbf{10})_2(\mathbf{10})_2$
$= (\mathbf{2^4 + 2^2})(\mathbf{11})_2(\mathbf{10})_2 + (\mathbf{2^2 + 1})(\mathbf{10})_2(\mathbf{10})_2$
$= (\mathbf{1111000})_2 + (\mathbf{10100})_2 = (\mathbf{10001100})_2$

3) Find the number of ways to put eight different books in five boxes, if no box is allowed to be empty? (This question is in section 4 that is not included, but)

$$5^8 - \binom{5}{1}.4^8 + \binom{5}{2}.3^8 - \binom{5}{3}.2^8 + \binom{5}{4}.1^8$$

Saudi Electronic University / College of Computing and Informatics
Discrete Mathematics (Math 150) / Assignment week 12

Section A

Determine whether each of these statements is true or false

1) Suppose that the relation **R** on a set is represented by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Then **R** is reflexive, symmetric but not anti-symmetric. **T**

2) The transitive closure of a relation **R** equals the connectivity relation $\boldsymbol{R}^* = \bigcup_{\boldsymbol{n=1}}^{\infty} \boldsymbol{R^n}$. T

3) The subsets $\{-3,-2,-1, 0\}$ and $\{0, 1, 2, 3\}$ are partitions of the set $\{-3,-2,-1, 0, 1, 2, 3\}$. **T**

4) The poset $(Z^+, |)$ is a lattice. **T**

5) Warshall's algorithm is an efficient method for computing the anti-symmetric closure of a relation. **F**

Section B

Each of the multiple choice objective questions contain four answers, of which one correct answer is to be marked.

1) Relations **R** are defined on the set {1,2,3,4}. Then, which of the following is false?

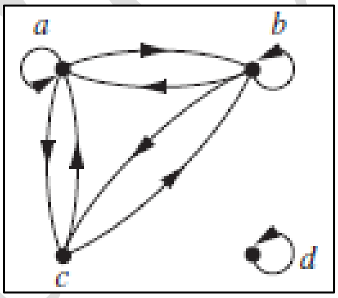
a) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} is reflexive, symmetric, antisymmetric and transitive.

b) {(2, 4), (4, 2)} is symmetric only.

c) {(1, 1), (2, 2), (3, 3), (4, 4)} is reflexive, symmetric, antisymmetric and transitive.

d) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)} is not reflexive, not symmetric, not anti-symmetric and not transitive.

2) The relation R represented by the directed graph is shown below



Then which of the following is true.

a) R is reflexive and symmetric.

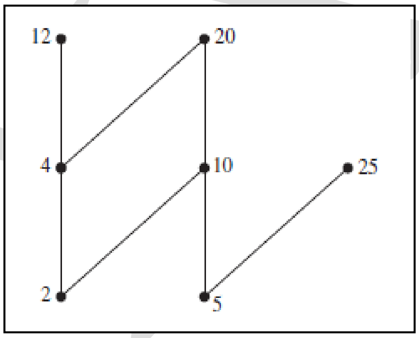
b) R is symmetric and transitive.

c) R is symmetric and anti-symmetric.

d) R is anti-symmetric and transitive.

3) Which of these relations on {0, 1, 2, 3} is a partial ordering?
a){(0, 0), (2, 2), (3, 3)}
b) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)}
c) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3), (3, 0), (3, 3)}
d) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)}

4) The Hasse diagram of the poset ({2, 4, 5, 10, 12, 20, 25}, |) is given below. Then maximal and minimal elements are



a) maximal elements are 12,20 and minimal elements are 2,5,25.
b) maximal elements are 25 and minimal elements are 2.
c) maximal elements are 12,20,25 and minimal elements are 2,5.
d) maximal elements are 20,25 and minimal elements are 2,5.

5) How many relations are there on a set with n elements?

a) $n!$ b) 2^{n^2} c) 2^n d) 2n.

Section C

Short answer type questions

1) Find the matrix representing the composition relation $S \circ R$, where the matrices representing relations R and S are

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2) Let m be an integer with m > 1. Show that the congruence modulo m relation **R** = {(a, b) | a ≡ b (mod m)} is an equivalence relation on the set of integers.

Testing if $\frac{(a-b)}{m} \in Z$ is equivalent.

1.1- Is it Reflexive?

a ≡ a (mod m) is true because m |(a-a). i.e. (a, a) ∈ R.

1.2- Is it symmetric?

If (a, b) ∈ R then a ≡ b (mod m), i.e. m |(a-b). implies m |(b-a)

So b ≡ a (mod m), hence (b,a) ∈ R.

1.3- Is it transitive?

If (a, b), (b,c) ∈ R then a ≡ b (mod m) and b ≡ c (mod m), i.e. m |(a-b) and m |(b-c). implies m |[(a-b)+ (b-c)].implies m |(a-c).

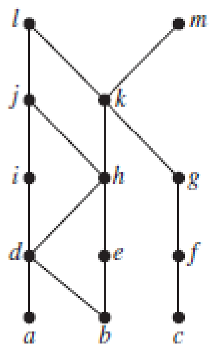
So a ≡ c (mod m), hence (a,c) ∈ R.

All 3 conditions are satisfied therefore it is an equivalence relation.

$\therefore$ ***So R = {(a, b) a ≡ b(mod m)}***
is an equivalence relation on the set of integers.

3) Draw the Hasse diagram for the "less than or equal to" relation on {1,2,3,4}.

4) Answer these questions for the partial order represented by this Hasse diagram.



a) Find the greatest element, if it exists.
There is no "greatest element" because we have more than one maximal.
b) Find the least element, if it exists.
There is no "least element" because we have more than one minimal.
c) Find all upper bounds of {a, b, c}.
(k, m, l) are all upper bounds of {a, b, c}.
d) Find the least upper bound of {a, b, c}, if it exists.
(k) is the least upper bounds of {a, b, c}.
e) Find all lower bounds of {f, g, h}.
There is no lower bound for {f, g, h}.
f) Find the greatest lower bound of {f, g, h}, if it exists.
No "greatest lower bound" exists because no lower bound exists.

5) Let **R** be the relation **R** = {(a, b) | a < b} on the set of integers.
What is the reflexive, symmetric closure of **R**?

Reflexive closure = R ∪ Δ = {(a, b) | a < b} ∪ {(a, a) | a ∈ Z} = {(a, b) | a ≤ b}.

Where Δ = {(a, a) | a ∈ Z}

Symmetric closure = R ∪ R^{-1} = {(a, b) | a < b} ∪ {(b, a) | a < b} = {(a, b) | a ≠ b}.

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Discrete Mathematics (Math 150) / Assignment week 13

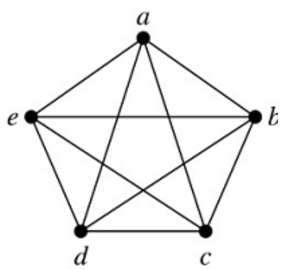
Section A

Determine whether each of these statements is true or false.

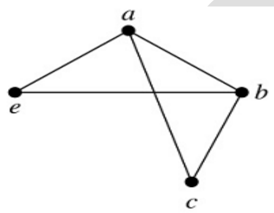
1. An undirected graph has an even number of vertices of odd degree. T
2. A complete graph on n vertices, denoted by Kn, is a simple graph that contains exactly one edge between each pair of distinct vertices. T
3. The simple graphs G1 = (V1; E1) and G2 = (V2; E2) are isomorphic if there is a one-to-one function from V1 to V2. F
4. The handshaking theorem is valid only for undirected graph. F
5. If adjacency matrix of a graph is a zero matrix then the graph is simple. T

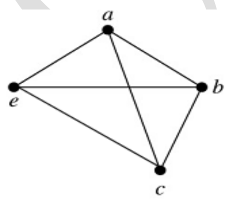
Section B

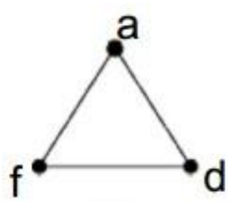
2. Each of the multiple choice objective questions contain three answers, of which one correct answer is to be marked.



(a) Which of these graphs is not sub graph of K5

A) 

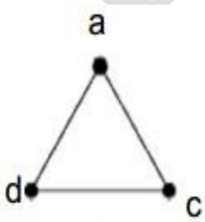
B) 

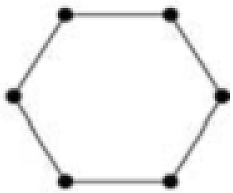
c) 

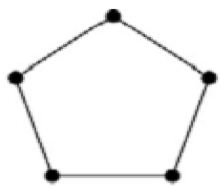
(b) In K4 Graph number of edges is

A) 10 B) 4 C) 6 D) 8

(c) Which of these graphs is Bipartite graph

A) 

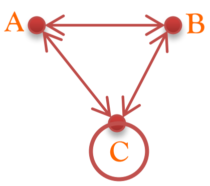
B) 

C) 

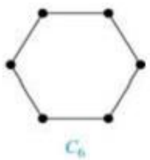
Section C

Solve the following:

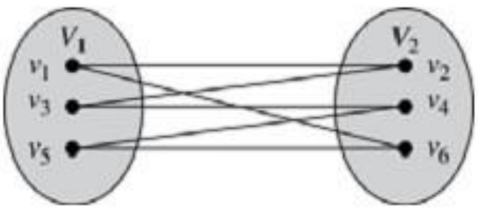
3. Given the adjacency matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ draw its corresponding graph.



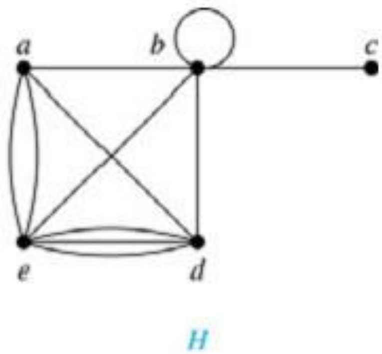
4. Show that C6 is bipartite.



C6 vertex set can be partitioned into the two sets Ver1 = {v1, v3, v5} and Ver2 = {v2, v4, v6}, every edge of C6 connects a vertex in Ver1 and a vertex in Ver2 so C6 is bipartite.



5. What are the degrees and neighborhoods of the vertices in the graphs H?



Neighborhood of (a) = {b, d, e}, <u>**deg (a) = 4**</u>

Neighborhood of (b) = {a, b, c, d, e}, <u>**deg (b) = 6**</u>

Neighborhood of (c) = {b}, <u>**deg (c) = 1**</u>

Neighborhood of (d) = {a, b, e}, <u>**deg (d) = 5**</u>

Neighborhood of (e) = {a, b, d}, <u>**deg (e) = 6**</u>

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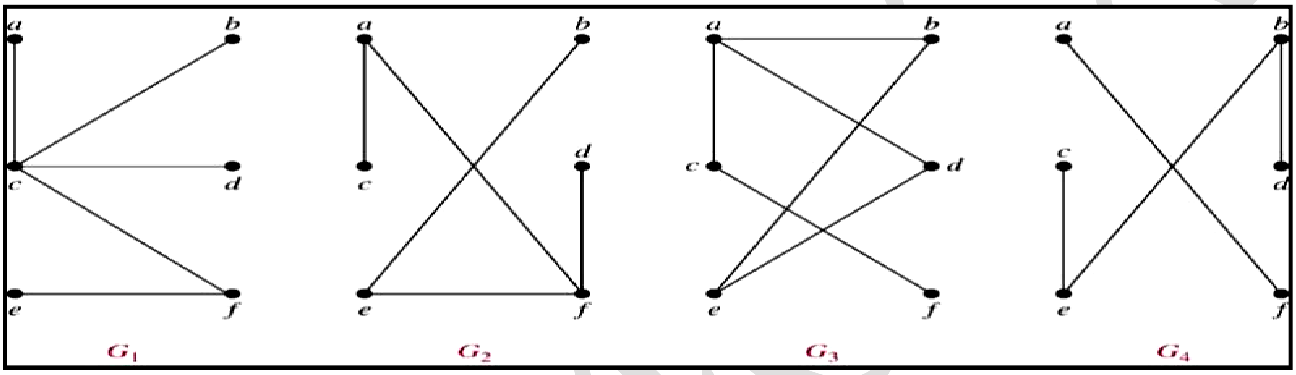
Assignment of week 14/ Math 150

Section 1: (True or False Questions)

1- A connected graph without any simple circuit is called a tree. T
2- A tree with n vertices has n-1 edges. T
3- If descendants and ancestors of a node are empty then this node has two children. F
4- A leaf is a vertex of degree 1. T
5- In a binary tree, the degree of root is 2^2. F

Section 2: (Multiple choice questions)

1 Which of the graphs are trees?



(a)G1G2 (b) G2G3 (c)G4 (d)None

2- The diameter of a tree is called.
e> Vertex
f> Edges
g> Radius
h> Longest path.

3- The vertex connectivity of any tree is.
b> two b> one c> three d> zero

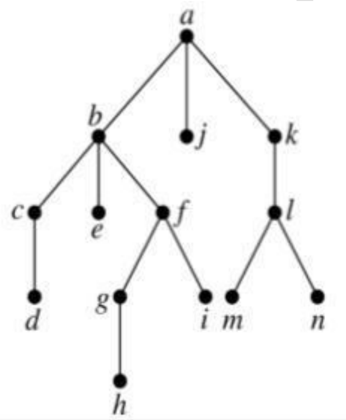
4- A tree with 99 edges has (.....) vertices.
b> 100 b> 98 c> 0 d> 97

5- Suppose that a full 4-ary tree has 100 leaves. How many internal vertices does it have?
b> 33 b> 23 c> -1 d> 10

Section 3

6. Define center of graph.

2. What is the height of the Tree and find the degrees of each <u>internal node</u> of the tree.



The height of the Tree is 4

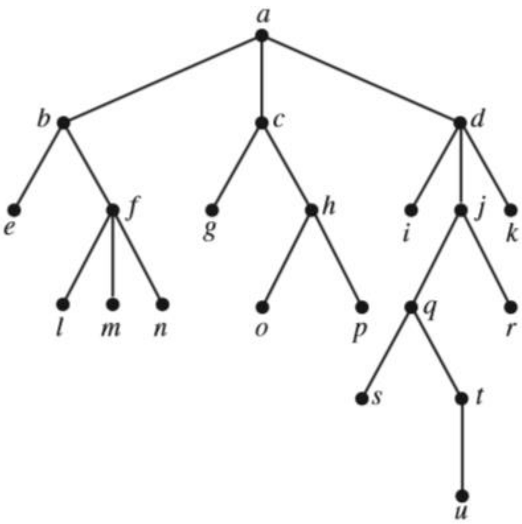
deg(a) =3 , deg(b) =4

deg(c) =2 , deg(f) =3

deg(k) =2 , deg(l) =3

deg(g) =2

3. Answer the questions for the following tree:



a) Which vertex is the root? (a)

b) Which vertices are internal? (a), (b), (c), (d), (f), (h), (j), (q), (t)

c) Which vertices are leaves? (e), (g), (i), (k), (l), (m), (n), (o), (p), (r), (s), (u)

d) Which vertices are children of j? (r), (q)

e) Which vertex is the parent of h? (c)

f) Which vertices are siblings of o? (p)

g) Which vertices are ancestors of m? (a), (b), (f)

h) Which vertices are descendants of b? (e), (f), (l), (m), (n)