Ministry of Higher Education **CSTS**

Kingdom of Saudi Arabia **SEU, KSA**

Discrete Mathematics (Math 150)

Level III, Assignment 4

(2016)

I. State whether the following statements are True or False **[6]**

(1) The relation $R = \left((1,1),(2,2),(1,2),(2,1),(3,3),(4,4)\right)$ on the set $A = \{1,2,3,4\}$ is antisymmetric. (1) **False**

(2) A relation on a set A is an equivalence relation if it is reflexive, antisymmetric, and transitive. (2) **False**

(3) The sum of degrees of the vertices of an undirected graph is odd.

(3) **False**

(4) The wheel W_6 has 6 edges. (4) **False**

(5) A tree is a connected undirected graph with no simple circuits. (5) **True**

(6) In a full binary tree with 1000 internal vertices, the edges are 2000.

(6) **True**

II. From questions 1 to 6 select the appropriate choice(s) as your response. **[6]**

(1) The relation R represented by the matrix $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is

A. Reflexive

B. Symmetric

C. Antisymmetric

D. Reflexive and Antisymmetric

(2) The number of relations on the set $A = \{1, 2, 3\}$

A. 8

B. 256

C. 512

D. 3

(3) For which value of *n*, the cycle C_n is bipartite?

A. 6

B. 7

C. 8

D. 9

(4) Let $G = (V, E)$ be a graph with 12 directed edges, then the sum of in-degrees of all vertices in G are

A. 6

B. 12

C. 24

D. 10

(5) A full 3-ary tree with 100 vertices has _______ leaves.

A. 10

B. 11

C. 12

D. 67

(6) A vertex of a rooted tree with no children is called

A. internal vertex

B. parent

C. leaf

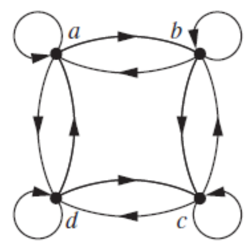
D. ancestor

III. Answer the following. Each question carries 3 Marks. **[3X6=18]**

1. If $R = \{(a,a),(a,b),(a,d),(b,a),(b,b),(b,c),(c,b),(c,c),(c,d),(d,a),(d,c),(d,d)\}$ represents a relation on the set $A = \{a,b,c,d\}$, then express the relation R by (i) matrix and (ii) directed graph. [3]

Solution: The matrix of the relation R is $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

The directed graph for the relation R is



2. Show that the relation $R = \{(a,b) \mid a - b \text{ is an integer}\}$ is an equivalence relation on the set of real numbers. [3]

Solution: As *a* - *a* = *0* is an integer implies that $(a,a) \in R$.

Therefore R is reflexive.

Let $(a,b) \in R$, then $a - b$ is an integer and $b - a$ is also an integer.

Implying that if $(a,b) \in R$ *then* $(b,a) \in R$.

Therefore R is symmetric.

Let $(a,b) \in R$ and $(b,c) \in R$, then $a - b$ and $b - c$ be are integers.

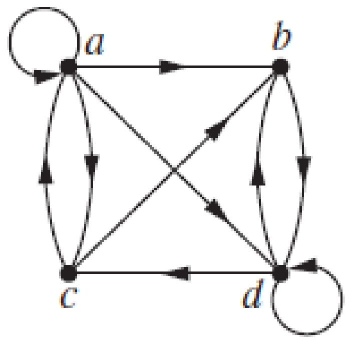
Now $(a - b) + (b - c) = a - c$ is also an integer.

Implying that if $(a,b) \in R$ *and* $(b,c) \in R \Rightarrow (a,c) \in R$.

Therefore R is transitive.

As R is reflexive, symmetric and transitive, the relation R is an equivalence relation.

3. Represent the following graph with an adjacency matrix and find the in-degrees and out-degrees of all the vertices. [3]



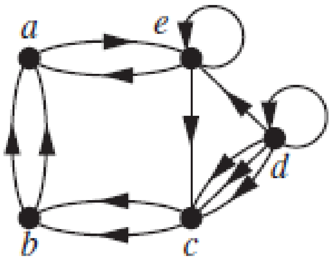
Solution:

The adjacency matrix for the given graph is $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

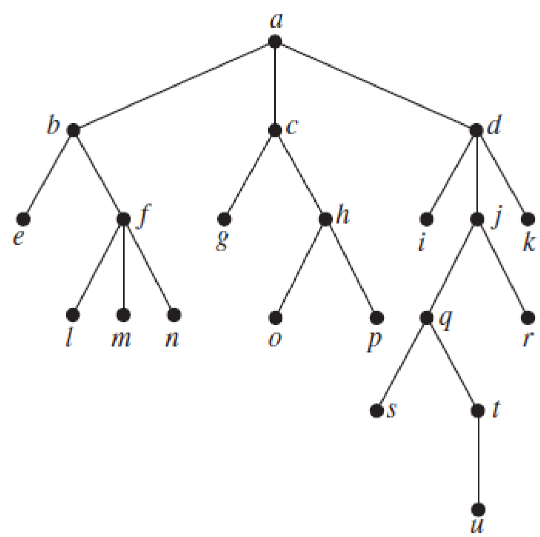
$$\deg^-(a) = 2, \deg^+(a) = 4, \deg^-(b) = 3, \deg^+(b) = 1,$$
$$\deg^-(c) = 2, \deg^+(c) = 2, \deg^-(d) = 3, \deg^+(d) = 3$$

4. Draw the graph represented by the adjacency matrix $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$ [3]

Solution: The graph for the given adjacency matrix is:



5. Answer the questions about the rooted tree illustrated. [3]



a) Which vertex is the root?

b) Which vertices are internal?

c) Which vertices are leaves?

d) Which vertices are siblings of *o*?

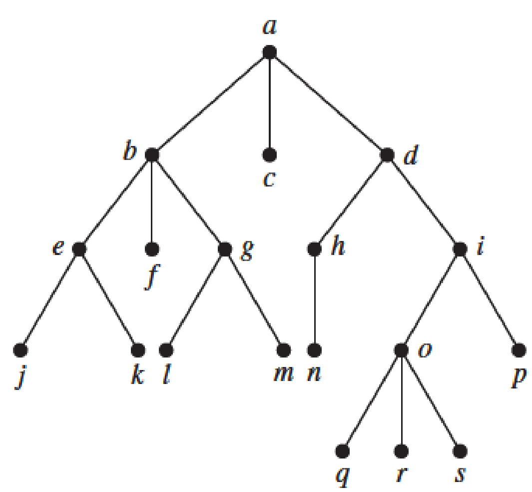
e) Which vertices are ancestors of *m*?

f) Which vertices are descendants of *b*?

Solution: **a)** *a* **b)** *a, b, c, d, f, h, j, q, and t* **c)** *e, g, i, k, l, m, n, o, p, r, s and u*

d) *p* **e)** *f, b and a* **g)** *e, f, l, m and n.*

6. Answer the questions about the rooted tree illustrated. [3]



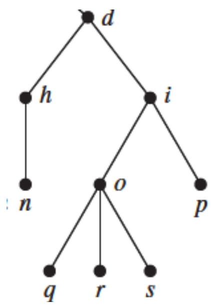
a) Is the above rooted tree is a full *m*-ary tree for some positive integer *m*?

b) What are the levels of the vertices *a*, *c*, *n and s*?

c) Draw a subtree rooted at the vertex *d*.

d) **Solution**: a) No b) 0, 1, 3 and 4

c)



******