Ministry of Higher Education **CSTS**
Kingdom of Saudi Arabia **SEU, KSA**

Discrete Mathematics (Math 150)
Level III, Assignment 3
(2016)

I. State whether the following statements are True or False **[6]**

(1) Principle of Mathematical Induction is used to prove propositional functions *P(n)* valid for negative integers *n*.

(1) **False**

(2) The well-ordering property states that every nonempty set of positive integers has a least element.

(2) **True**

(3) $C(5,0) = 0$

(3) **False**

(4) The number of three digit number formed from the set {1, 2, 3, 4} when repetition is allowed any number of times is 12.

(4) **False**

(5) The recurrence relation $a_n = a_{n-1}^2 + a_{n-2}$ is linear homogeneous with constant coefficients.

(5) **False**

(6) The generating function for the finite sequence 1, 4, 16, 64 and 256 is $1-4x+16x^2-64x^3+256x^4$.

(6) **False**

II. Select one of the alternatives from the following questions as your answer. [6]

(1) Principle of Mathematical Induction, expressed as a rule of inference for the domain of positive integers is $\left(P(x) \wedge \forall k(P(k) \rightarrow P(k+1))\right) \rightarrow \forall n P(n)$, then x is:

A. 0

B. 1

C. -1

D. ½.

(2) If *f* is defined recursively by $f(n+1) = 3f(n)^2 - 4f(n-1)^2$ with $f(0) = -1$, $f(1) = 2$ for positive integers n, then $f(2)$ is:

A. 16

B. -8

C. 8

D. -16

(3) A drawer contains half a dozen black socks and half a dozen brown socks, all unmatched. A man takes socks out at random in the dark, the number of socks he must take out to be sure that he has at least two black socks?.

A. 3

B. 4

C. 6

D. 8

(4) If $C(n, r) = 35$ and $P(n, r) = 210$, the values of n and r are:

A. 7, 3

B. 3, 7

C. 5, 5

D. 7, 4

(5) The characteristic equation of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$ is obtained by substituting

A. $a_n = r^2$

B. $a_n = r^{-2}$

C. $a_n = r^n$

D. $a_n = r^{-n}$

(6) The roots of the characteristic equation $r^2 - 4r + 4 = 0$ for the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$ are $r = 2, 2$. The solution of the recurrence relation is:

A. $a_n = c_1 2^n + c_2 2^n$

B. $a_n = c_1 2^{-n} + c_2 2^{-n}$

C. $a_n = c_1 2^n + c_2 2.2^n$

D. $a_n = c_1 2^n + c_2 n.2^n$

III. Answer the following. Each question carries 3 Marks. **[6 X 3 = 18]**

1. Prove by Principle of Mathematical Induction:

$1.2.3 + 2.3.4 + \ . \ . \ . + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$ for all positive integers *n*. [3]

Solution: Let $P(n) = 1.2.3 + 2.3.4 + \ . \ . \ . + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Basis Step: Let *n* = 1

L.H.S. = 1.2.3 = 6

R.H.S. = 1(1+1)(1+2)(1+3) / 4 = 24/4 = 6

As L.H.S. = R.H.S

$\Rightarrow P(1)$ is true.

Inductive Step: Assume that *P*(*k*) is true.

That means $1.2.3 + 2.3.4 + \ . \ . \ . + k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4}$ …..……(1)

for some arbitrary number *k*.

To prove that *P*(*k* + 1) is true.

That means to prove that:

$$1.2.3+2.3.4+\ ...\ +(k+1)(k+1+1)(k+1+2)=\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}$$

$$\Rightarrow 1.2.3+2.3.4+\ ...\ +(k+1)(k+2)(k+3)=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

Consider L.H.S.

$$1.2.3+2.3.4+\ ...\ +(k+1)(k+2)(k+3)$$
$$\Rightarrow \{1.2.3+2.3.4+\ ...+(k)(k+1)(k+2)\}+(k+1)(k+2)(k+3)$$
$$\Rightarrow \frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)\quad (\text{since } from\ (1))$$
$$\Rightarrow \frac{k(k+1)(k+2)(k+3)+4(k+1)(k+2)(k+3)}{4}$$
$$\Rightarrow \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

R.H.S. = $\frac{(k+1)(k+2)(k+3)(k+4)}{4}$

As L.H.S. = R.H.S.

$\Rightarrow P(k + 1)$ is true.

By Principle of Mathematical Induction, the statement $P(n)$ is true for all positive integers n.

2. Give a recursive definition of

(i) The set of positive integers that are multiples of 5.

(ii) The set of even integers. [3]

Solution: (i) Let S be the subset of integers recursively defined by

Basis Step: $5 \in S$.

Inductive step: If $x \in S$ and $y \in S$, then $x + y \in S$.

(ii) Let S be the subset of integers recursively defined by

Basis Step: $0 \in S$.

Inductive step: If $x \in S$, then $x+2 \in S$ and $x-2 \in S$.

3. In a University, there are three women and four men working as faculty in Mathematics department. Find the number of ways to select a committee of three members of the department if at least one woman must be on the committee. [3]

Solution: The possible selections are:

Women(3)	Men(4)	Selection(3)
1	2	${}^{3}C_{1}\times{}^{4}C_{2}=\frac{3!}{1!\,2!}\times\frac{4!}{2!\,2!}=18$
2	1	${}^{3}C_{2}\times{}^{4}C_{1}=\frac{3!}{2!\,1!}\times\frac{4!}{1!\,3!}=12$
3	0	${}^{3}C_{3}\times{}^{4}C_{0}=\frac{3!}{3!\,0!}\times\frac{4!}{0!\,4!}=1$

The number of ways of selecting a committee of three members of the department so that at least one woman is there on the committee = 18 + 12 + 1 = 31 ways.

4. Find the coefficient of x^7y^3 in the expansion of $(3x-5y)^{10}$. [3]

Solution:

$$(3x-5y)^{10}=[(3x)+(-5y)]^{10}=\sum_{r=0}^{10}\binom{10}{r}(3x)^{10-r}(-5y)^{r}$$

$$=\sum_{r=0}^{10}\binom{10}{r}(3)^{10-r}(-5)^{r}(x)^{10-r}(y)^{r}$$

To find the coefficient of x^7y^3, put r = 3.

Therefore, the coefficient of x^7y^3 in the expansion of $(3x-5y)^{10}$ is $\binom{10}{3}3^7(-5)^3$.

5. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2,\ a_0 = 1,\ a_1 = 2$. [3]

Solution: Given $a_n = 5a_{n-1} - 6a_{n-2}$ ....(1)

Degree = 2.

Put $a_n = r^n$ in the given recurrence relation (1).

$$\Rightarrow r^n = 5r^{n-1} - 6r^{n-2}$$
$$\Rightarrow r^2 - 5r + 6 = 0$$
$$\Rightarrow (r-2)(r-3) = 0$$

The roots of the characteristic equation are $r = 2$, $r = 3$.

The solution of (1) is $a_n = c_1(2)^n + c_2(3)^n$ ....(2)

Given, $a_0 = 1 \Rightarrow c_1 + c_2 = 1$ ....(3)

$a_1 = 2 \Rightarrow 2c_1 + 3c_2 = 2$ ....(4)

Solving equations (3) and (4), we get $c_1 = 1,\ c_2 = 0$.

The solution of the given recurrence relation is $a_n = (2)^n$.

6. Find the values of the extended binomial coefficients $\binom{-3}{4}$ *and* $\binom{1/3}{4}$. [3]

Solution: If *n* is a real number and *r* is a nonnegative integer, then

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ .\ .\ .\ (n-r+1)}{r!}$$

$$\binom{-3}{4} = \frac{(-3)(-3-1)(-3-2)(-3-3)}{4!} = 15.$$

$$\binom{1/3}{4} = \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)\left(\frac{1}{3}-3\right)}{4!} = \frac{-10}{243}.$$

*****