Kingdom of Saudi Arabia
Ministry of Higher Education
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المملكة العربية السعودية
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DISCRETE MATHEMATICS (MATH-150)

Level III

ASSIGNMENT-2 (2016)

Section-I

1. State whether the following statements are True or False **(6×1= 6 Marks)**

a) If $|A| = 7, |B| = 5, |A \cap B| = 2$ then $|A \cup B| = 12$.

(a) **False**

b) If A = {a, b, c} and B = {b, {c}} then c ∈ A – B.

(b) True

c) If $f : Z \to Z$ defined by $f(x) = x + 1$ then f is not invertible.

(c) **False**

d) Greedy Algorithms can be used to solve optimization problems.

(d) **True**

e) $4 +_{10} 12$ is equal to 6.

(e) **False**

f) The Binary expansion of $(64)_{10}$ is $(1000000)_2$.

(f) **True**

Section-II

2. Select one of the alternative form each of the following questions as your answer (6×1= 6 Marks)

(a) If U= {1, 2, 4, 7, 8, 9}, A={1,2,4,7} and B={4,7,8,9}, then $A \cap \overline{B} =$

(A) {1, 2, 4, 7}

(B) {1, 2}

(C) {8, 9}

(D) {}

(b) If $f(x) = x^2 - 3$ and $g(x) = x + 7$ then $(f \circ g)(x)$ is

(A) x^2

(B) $2x + 1$

(C) $x^2 + 4$

(D) $x^3 + 2$

NOTE: None of the alternative is correct. Award one mark for attempting the question.

(c) The value of $\sum_{i=2}^{6}(-1)^i$ is

(A) 0

(B) 2

(C) -1

(D) 1

(d) To sort a list with n elements, the insertion sort begins with which of the following element?

(A) First

(B) Second

(C) Third

(D) Fourth

(e) If a and b are positive integers, then

(A) $\gcd(a,b) \cdot lcm(a,b) = a/b$

(B) $\gcd(a,b) \cdot lcm(a,b) = b/a$

(C) $\gcd(a,b) \cdot lcm(a,b) = 1/ab$

(D) $\gcd(a,b) \cdot lcm(a,b) = a \cdot b$

(f) The prime factorization of 666 is

(A) $2^2 \cdot 5^2$

(B) $2^2 \cdot 3^2$

(C) $2^2 \cdot 3^2 \cdot 5^2$

(D) $2 \cdot 3^2 \cdot 37$

Section-III

Answer the following. Each carries 3 marks. **(6×3=18 Marks)**

3. If $f : Z^+ \to Z^+$ defined by $f(x) = 2x + 3$, then show that f is an injective function.

Solution: To prove that a function f is injective we need to show that if $f(a) = f(b)$ for arbitrary a, b in the domain Z^+ with ($a \neq b$), then a = b.

Let $f(a) = f(b)$

$\Rightarrow 2a + 3 = 2b + 3$ (by definition of f)

$\Rightarrow 2a = 2b$

$\Rightarrow a = b$

$\Rightarrow f$ is an injective function .

4. Consider the recurrence relation $f_n = f_{n-1} + 3f_{n-2}$ with initial conditions $f_0 = 0$, $f_1 = 1$. Find f_3, f_5, and f_7 .

Solution: Given $f_n = f_{n-1} + 3f_{n-2}$ and $f_0 = 0$, $f_1 = 1$.

Putting n = 1, 2, 3, 4, 5, 6 and 7 in the given recurrence relation, we get

$$f_2 = f_1 + 3f_0 = 1 + 3(0) = 1$$
$$f_3 = f_2 + 3f_1 = 1 + 3(1) = 4$$
$$f_4 = f_3 + 3f_2 = 4 + 3(1) = 7$$
$$f_5 = f_4 + 3f_3 = 7 + 3(4) = 19$$
$$f_6 = f_5 + 3f_4 = 19 + 3(7) = 40$$
$$f_7 = f_6 + 3f_5 = 40 + 3(19) = 97$$

5. Prove the distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for three sets A, B and C.

Solution: One can prove this identity by showing that each side is a subset of the other side

(OR)

By using a membership table for the combinations on both sides.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

As the columns for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are same, the identity is valid.

6. Use the bubble sort and put the list 7, 8, 4, 6, 5 in increasing order.

Solution:

First Pass: $(7,8,4,6,5) \to (7,4,8,6,5) \to (7,4,6,8,5) \to (7,4,6,5,8)$

Second Pass: $(7,4,6,5,8) \to (4,7,6,5,8) \to (4,6,7,5,8) \to (4,6,5,7,8)$

Third Pass: $(4,6,5,7,8) \to (4,5,6,7,8)$

Fourth Pass: $(4,5,6,7,8)$

7. Find the octal expansion of $(10\ 1110\ 1011\ 1101)_2$.

Solution: To convert the binary expansion to octal, group the binary digits into blocks of three and write the corresponding octal digits. Add 0's if necessary at the beginning to form a block of three binary digits.

Hence $(010\ \ 111\ \ 010\ \ 111\ \ 101)_2 = (\ 2\ 7\ 2\ 7\ 5)_8$

8. Find the greatest common divisor of 414 and 662, using the Euclidean algorithm.

Solution: By successively applying the division algorithm, we get the following:

662 = 414.1 + 248

414 = 248.1 + 166

248 = 166.1 + 82

166 = 82.2 + 2

82 = 2.41 + 0

Hence, gcd(662, 414) = the last nonzero remainder in the sequence of divisions.

i.e., gcd(662, 414) = 2.

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