Ministry of Higher Education
Kingdom of Saudi Arabia

CSTS
SEU, KSA

Discrete Mathematics (Math 150)
Level III, Assignment 1
(2015)

1. State whether the following statements are true or false: [9]

   (a) $y \vee \neg(\neg x \wedge y)$ is a tautology.

   (a) True

   (b) The compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are not equivalent.

   (b) False

   (c) The negation of conjunction of two propositions is equivalent to the disjunction of the negation of those propositions.

   (c) True

   (d) In Boolean algebra, $1 + x = x$.

   (d) False

   (e) The product of sums is basically the ORing of ANDed terms.

   (e) False

   (f) The dual of $(x + y).z$ is $x.y + z$.

   (f) True

   (g) The universal quantification $\forall x P(x)$ is false only when $P(x)$ is false for each value of x in the domain.

   (g) False

   (h) The quantified statement $\forall x, y \in \mathbb{N},\ x + y > \ x \ \wedge \ x + y > \ y$ is true.

   (h) True

   (i) Negation of $\exists x(p(x) \wedge q(x))$ is $\forall x \neg p(x) \rightarrow q(x)$.

   (i) False

2. Select one of the alternatives from the following questions as your answer. [9]

   (a) The compound proposition $p \wedge q$ is equivalent to

      A. $p \rightarrow q$

      B. $\neg(p \rightarrow \neg q)$

      C. $\neg p \rightarrow q$

      D. None

   (b) The converse statement of the compound proposition $q \rightarrow p$ is

      A. $\neg p \rightarrow \neg q$

      B. $\neg q \rightarrow \neg p$

      C. $p \rightarrow q$

      D. None

   (c) The compound proposition $p \vee (p \wedge q)$ is equivalent to

      A. p

      B. q

      C. $p \vee q$

      D. $p \wedge q$

   (d) Which of the following expression is in the sum-of-products form?

      A. $(x + y).(z + w)$

      B. $x.y + z.w$

      C. $(x)y(zw)$

      D. None of above

   (e) The values of x, y, z and w that makes the sum term $\bar{x} + y + \bar{z} + w$ equal to zero is

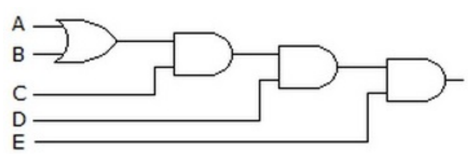
      A. $x = 1,\ y = 0,\ z = 0,\ w = 0$

      B. $x = 0,\ y = 1,\ z = 0,\ w = 0$

      C. $x = 1,\ y = 0,\ z = 1,\ w = 1$

      D. $x = 1,\ y = 0,\ z = 1,\ w = 0$

   (f) The resulting Boolean expression for the following circuit is



A. $(C(A+B)D)+E$
B. $[(C(A+B)D)]E$
C. $ABCDE$
D. $C(A+B)DE$

(g) The multiplicative inverse law for the nonzero rational numbers is given by
A. $\forall x \exists y\ (xy=1)$
B. $\exists x \forall y\ (xy=1)$
C. $\forall x \forall y\ (xy=1)$
D. all of the above.

(h) The negation of $\exists x(p(x) \wedge q(x))$ is
A. $\forall x,\ p(x) \to \neg q(x)$
B. $\forall x,\ \neg p(x) \to q(x)$
C. $\forall x,\ p(x) \vee \neg q(x)$
D. $\forall x,\ p(x) \to q(x)$

(i) When multiple quantifiers differ, then the meaning of a predicate logic sentence
A. is determined by arithmetic operators
B. depends on order
C. is ambiguous
D. is independent of order

3. Construct the truth table of compound proposition $(p \to q) \wedge (\neg p \to r)$. [2]

Solution: The truth table is given below

p	q	$p \to q$	$\neg p$	r	$\neg p \to r$	$(p \to q) \wedge (\neg p \to r)$
T	T	T	F	T	F	F
T	F	T	F	F	T	T
F	T	F	T	F	T	F
F	F	T	T	T	T	T

4. Find the contrapositive and inverse statements of the statement "If you do every exercise in this book then you are a good student" [2]

Solution: Let us suppose that p denotes the proposition you do every exercise in this book and q denotes the proposition you are a good student. Then $\neg p$ will denotes the proposition you don't do every exercise in this book and $\neg q$ denotes the proposition you are not a good student.

We know that the contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$. Therefore the contrapositive of the given statement will be "If you are not a good student then don't do every exercise in this book".

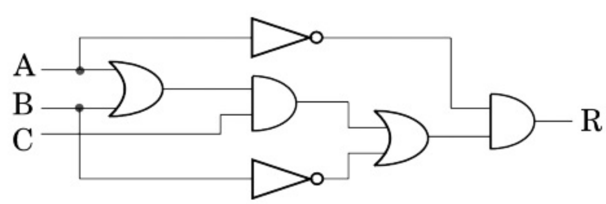
The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$. Therefore the inverse of the given statement will be "If you don't do every exercise in this book then you are not a good student."

5. Find the values of the boolean function $f(x, y, z) = \bar{x}\bar{y}z + \bar{x}yz + xy\bar{z}$. [2]

Solution: The values of the Boolean function are tabulated as follows:

x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	1
0	1	0	0
1	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1
1	1	1	0

6. Determine the output of the following combinatorial circuit: [2]



Solution:

The resulting output of the given circuit will be

$$R = \bar{A}.(\bar{B} + C.(A + B)) \qquad \text{(Verify it.)}$$

7. Prove that $n^2 - 2$ is not divisible by 5. [2]

Solution: We will prove that n^2-2 is not divisible by 5, by giving the counter example. Take $n = 6$, $n^2 - 2 = 34$, not divisible by 5.

8. Let the domain for x be the set of integers. [2]
$p(x)$: x is even.
$q(x)$: x is a prime number.
$r(x)$: 5 divides x.

Write the following symbols as quantified statements.

1. $\exists\ x, (p(x) \wedge q(x))$.

2. $\forall\ x(p(x) \wedge q(x)) \to r(x)$.

Solution: The quantified statements will be as follows:

1. Some integers are even and prime numbers.

2. 5 divides every even and prime integers.