Ministry of Higher Education
Kingdom of Saudi Arabia

CSTS
SEU, KSA

Discrete Mathematics (Math 150)
Level III, Assignment 4
(2015)

1. State whether the following statements are True or False [9]

(a) If a relation is reflexive, then it is an equivalence relation. (a) False

(b) The relation $R = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$ on the set $A = \{1,2,3,4\}$ is antisymmetric (b) True

(c) A set with a partial order relation is called Poset. (c) True

(d) A graph with no loops and multiple edges is called a simple graph. (d) True

(e) An undirected graph has an even number of vertices of even degree. (e) False

(f) In an undirected graph, the loop at a vertex contributes twice to the degree of that vertex. (f) True

(g) A tree is a simple graph. (g) True

(h) A rooted tree is a binary tree if every internal vertex has two children. (h) True

(i) Vertices that have children are called external vertices. (i) False

2. Select one of the alternatives from the following questions as your answer. [9]

(a) Let $R = \{(a,b) \mid a = b \text{ or } a = -b\}$ be a relation on the set of integers. Then R

A. Reflexive

B. Symmetric

C. Transitive

D. Equivalence

(b) If every vertex of a directed graph has a loop, then the relation associated with it is

A. Reflexive

B. Symmetric

C. Transitive

D. Equivalence

(c) The relation on a set represented by the matrix $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ is

A. Reflexive

B. Symmetric

C. Transitive

D. Equivalence

(d) The number of edges in the wheel W_3 are

A. 3

B. 4

C. 5

D. 6

(e) Two simple graphs $G_1 = (V_1, E_1) and G_2 = (V_2, E_2)$ are said to be isomorphic, if there is

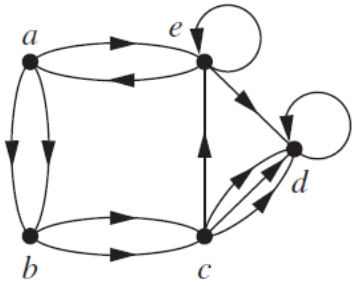
A. One-One function from $V_1 \to V_2$

B. Onto function from $V_1 \to V_2$

C. Bijective function $V_1 \to V_2$

D. Relation from $V_1 \to V_2$

(f) The in-degree and the out-degree of the vertex d in the following graph are:



A. 6 and 2

B. 5 and 3

C. 5 and 1

D. 5 and 2

(g) How many edges does a tree with 1000 vertices have?

A. 10

B. 100

C. 999

D. 99

(h) How many leaves does a full 3-ary tree with 10 vertices have?

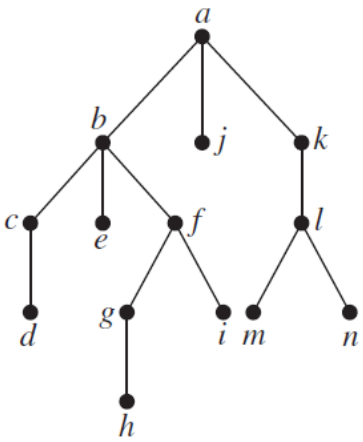
A. 9

B. 7

C. 10

D. 8

(i) What is the height of the tree in the following figure?



A. 5

B. 6

C. 4

D. 7

3. Let $R = \{(a,b) \mid a\ divides b\}$ be a relation on the set of positive integers. Show that R is not an equivalence relation. [2]

Solution:

As $a|a \Rightarrow (a,a) \in R$. Therefore R is reflexive.

But

$a \mid b\ does\ not\ mean\ that\ b \mid a\ always$
$i.e. (a,b) \in R\ and\ (b,a) \notin R$

Therefore R is not symmetric.

Hence R is not an equivalence relation.

4. Let $A = \{1,2,3\}$. Write a relation R on this set which is reflexive and antisymmetric. [2]

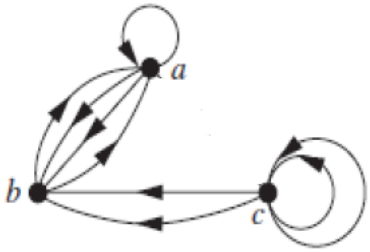
Solution: $R = \{(1,1),(2,2),(3,3),(1,2),(1,3),(2,3)\}$.

5. Draw the graph represented by the following adjacency matrix by using the ordering of vertices a, b, c, d . [2]

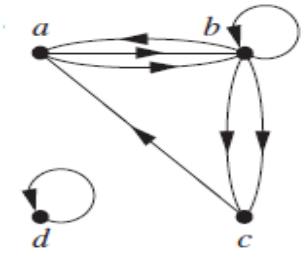
$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

Solution:

The graph with the given adjacency matrix is



6. Find the in-degree and out-degree of each vertex given in the following graph. [2]



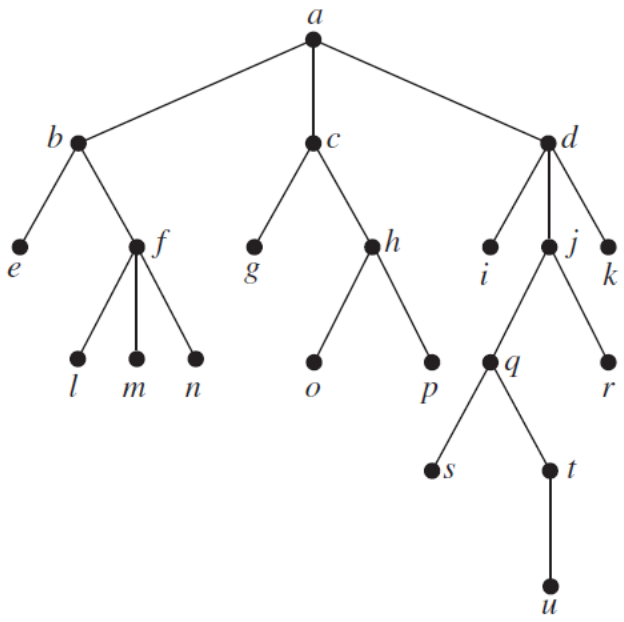
Solution: $\deg^-(a) = 2, \deg^+(a) = 2$

$\deg^-(b) = 3, \deg^+(b) = 4$

$\deg^-(c) = 2, \deg^+(c) = 1$

$\deg^-(d) = 1, \deg^+(d) = 1$

7. Answer the following questions about the rooted tree illustrated. [4]



(a) Which vertex is root?
(b) Which vertices are internal?
(c) Which vertices are leaves?
(d) Which vertices are children of *j* ?
(e) Which vertex is the parent of *h* ?
(f) Which vertices are siblings of *o* ?
(g) Which vertices are ancestors of *m* ?
(h) Which vertices are descendants of *b* ?

Solution:

(a) a
(b) a, b, c, d, f, h, j, q, t
(c) e, l, m, n, g, o, p, i, k, s, r, u
(d) q, r
(e) c
(f) p
(g) f, b, a
(h) e, f, l, m, n

(End of Assignment)