Ministry of Higher Education
Kingdom of Saudi Arabia

CSTS
SEU, KSA

Discrete Mathematics (Math 150)
Level III, Assignment 2
(2015)

1. State whether the following statements are true or false: [9]

   (a) Two sets are equal if and only if they have the same number of elements.

   (a) False

   (b) If A is a proper subset of B then the cardinality of A and B are same.

   (b) False

   (c) If two sets are disjoint then their intersection is not an empty set.

   (c) False

   (d) If range and codomain of a function f are equal then f is a surjective function.

   (d) True

   (e) If f is not bijective then f can not be invertible.

   (e) True

   (f) In an algorithm, infinite number of steps may be used to get the desired output.

   (f) False

   (g) If a 24-hour clock read 18:00, then after 95 hours it will read 17:00.

   (g) True

   (h) Every prime number is not even.

   (h) False

   (i) The integers 12, 17 and 21 are pairwise relatively prime number.

   (i) False

2. Select one of the alternatives from the following questions as your answer. [9]

(a) If $A = \{1, 7, 9, 15\}$ then the number of possible subsets of A are

A. 12

B. 16

C. 8

D. 6

(b) If $A = \{1, 3, 5, 7, 9, \ldots\}$ with the set of positive integers as the universal set. Then $\overline{A}$ is

A. Set of positive even integers

B. Set of negative even integers

C. Set of negative integers

D. None

(c) If $f : A \to B$ be a function such that $f(a) = 4,\ f(b) = 1,\ f(c) = 2$ and $f(d) = 5$ where $A = \{a, b, c, d, \}$ and $B = \{1, 2, 3, 4, 5, 6\}$ then

A. f is 1-1 and onto both.

B. f is onto but not 1-1.

C. f is 1-1 but not onto.

D. f is not a function.

(d) If $f : A \to B,\ g : B \to B$ and $h : B \to A$ are functions then composition function gof be a function from

A. $A \to A$

B. $B \to A$

C. $B \to B$

D. $A \to B$

(e) If $f : \mathbb{Z} \to \mathbb{Z}$ be a function such that $f(x) = 2x$, then

A. f is 1-1.

B. f is onto

C. f is bijective

D. All of the above

(f) The value of $\sum_{j=4}^{7} (-1)^j$ is

A. 0

B. 1

C. -1

D. 2

(g) Which of the following is NOT true:

A. $12 \equiv 4(\mod 2)$

B. $17 \equiv 7(\mod 10)$

C. $104 \equiv 91(\mod 12)$

D. $210 \equiv 10(\mod 5)$

(h) The value of the modular expression $(7 \ +_{11} \ 10) \cdot_{11} (9 \ +_{11} \ 6)$

A. 255

B. 2

C. 4

D. 6

(i) If $G.C.D(a,b) = 10$ and $ab = 40$ then $L.C.M(a,b) =$

A. 400

B. $\frac{1}{4}$

C. 4

D. 1

3. If $A = \{1,2,3,7,9,11,14,15,16\}$, $B = \{1,7,11,13,19\}$ and $C = \{2,4,6,8,10,11,12,19\}$, then show that [2]

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Solution: Here

$$\begin{aligned}
(A \cup B) =& \{1,2,3,7,9,11,13,14,15,16,19\} \\
(A \cup C) =& \{1,2,3,4,6,7,8,9,10,11,12,14,15,16,19\} \\
(A \cup B) \cap (A \cup C) =& \{1,2,3,7,9,11,14,15,16,19\} \\
(B \cap C) =& \{11,19\} \\
& \text{Therefore} \\
A \cup (B \cap C) =& \{1,2,3,7,9,11,14,15,16,19\}
\end{aligned}$$

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

4. If $f,\ g : \mathbb{Z} \to \mathbb{Z}$ be functions such that $f(x) = 3x + 2$ and $g(x) = 2x^2 + 1$, then find the values of $(gof)(x)$ and $(fog)(x)$. [2]

   Solution: By the definition of composition of function,

$$\begin{aligned}(gof)(x) =& g(f(x))\\ =& g(3x+2)\\ =& 2(3x+2)^2 + 1 \qquad \text{by definition of } g\\ =& 18x^2 + 24x + 9\\ (gof)(x) =& 18x^2 + 24x + 9.\end{aligned}$$

   Similarly,

$$\begin{aligned}(fog)(x) =& f(g(x))\\ =& f(2x^2+1)\\ =& 3(2x^2+1) + 2 \qquad \text{by definition of } f\\ =& 6x^2 + 5\\ (fog)(x) =& 6x^2 + 5.\end{aligned}$$

5. If $a_n = a_{n-1} + a_{n-2}$ with $a_0 = 1$, and $a_1 = 2$, then find the value of a_5 of the given sequence. [2]

   Solution: Since $a_n = a_{n-1} + a_{n-2}$ with $a_0 = 1$, and $a_1 = 2$, first find $a_2,\ a_3$ and a_4 then use them to find a_5.
   Put $n = 2$ in given expression for getting a_2, so

$$\begin{aligned}a_2 =& a_1 + a_0 = 1 + 2 = 3\\ a_3 =& a_2 + a_1 = 3 + 2 = 5\\ a_4 =& a_3 + a_2 = 5 + 3 = 8\\ a_5 =& a_4 + a_3 = 8 + 5 = 13\end{aligned}$$

   Therefore $a_5 = 13$.

6. List all the steps used to search for 9 in the sequence 1,3,4,5,6,8,9,11 using linear search.

Solution: Linear Search: Let $x = 9$. Now compare x with each element of the sequence. First element of the sequence $a_1 = 1$, since $x \neq 1$, so $x \neq a_1$. Compare it with $a_2 = 3$. Since $x \neq a_2$, compare it with $a_3 = 4$.
Similarly, compare x with successive elements of the sequence and stopped at $x = a_7 = 9$. Therefore the solution is the 7^{th} term of the sequence.

7. Find the octal expansion of $(765)_{10}$. [2]

Solution: First divide 765 by 8 to obtain

$$765 = 95 \times 8 + 5$$

Successively dividing quotients by 8, we get

$$\begin{aligned} 95 =& 11 \times 8 + 7 \\ 11 =& 1 \times 8 + 3 \\ 1 =& 0 \times 8 + 1 \end{aligned}$$

The successive remainder that we have found 5,7, 3 and 1 are the digits from the right to the left of 765 in base 8. Hence

$$(765)_{10} = (1375)_8.$$

8. Find the greatest common divisor of 314 and 520 by using the Euclidean Algorithm. [2]

Solution: Successive use of division algorithm gives

$$\begin{aligned} 520 =& 314 \times 1 + 206 \\ 314 =& 206 \times 1 + 108 \\ 206 =& 108 \times 1 + 98 \\ 108 =& 98 \times 1 + 10 \\ 98 =& 10 \times 9 + 8 \\ 10 =& 8 \times 1 + 2 \\ 8 =& 2 \times 4 + 0 \end{aligned}$$

Hence the GCD is the last nonzero remainder in the sequence of divisions. So

$$G.C.D(520, 314) = 2$$

 End of Assignment.