Representing Numerical Data

(Chapter 5)

Signed-Integer Representation:

- No obvious direct way to represent the sign in binary notation, options:
  - Sign-and-magnitude representation
  - 1's complement
  - 2's complement (most common)

Sign-and-Magnitude:

- Use left-most bit for sign
  - 0 = plus; 1 = minus
- Total range of integers the same
  - Half of integers positive; half negative
  - Magnitude of largest integer half as large
- Example using 8 bits:
  - Unsigned: 1111 1111 = +255
  - Signed: 0111 1111 = +127 and 1111 1111 = – 127
  - Note: 2 values for 0: +0 (0000 0000) and -0 (1000 0000)
- Example: Solve the following problem 13 – 6 using the Sign and Magnitude method:
  - Unsigned: $13_{10}=1101_2$ and $6_{10}=110_2$
  - Signed: $+13_{10}=0(1101)_2$ and $-6_{10}=1(0110)_2$
  - Note we use addition:

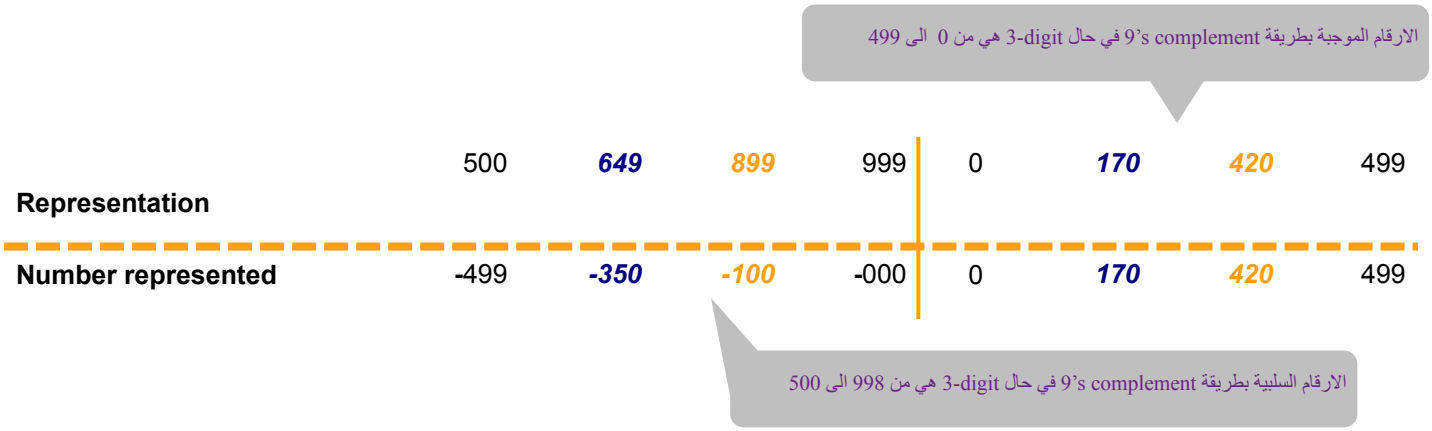
نضيف اصفار على يسار العدد لتكوين 4 bits او 8 bits بعد ذالك نضيف اشارة العدد 0 للموجب و 1 للسالب

13	=	0	1	1	0	1
+ (-6)	=	1	0	1	1	0
		1	0	1	1	1

- Now consider 6 − 5. Direct subtraction yields 0110 − 0101 = 0001. However, if we express it as 6 + (−5) and carry out the addition, we have 0110 + 1101 = 10011 and so the 4-bit sum word is 0011 (due to the 4-bit word length). Since 0011 not equal 0001, the Requirement is not satisfied.

9's Complement:

- *Taking the complement*: subtracting a value from a standard basis value
- Decimal (base 10) system diminished radix complement
  - Radix minus 1 = 10 – 1 ➨ 9 as the basis
  - 3-digit example: base value = 999
  - Range of possible values 0 to 999 arbitrarily split at 500



- Example:
  - The sign-and-magnitude value of 3-digit number 899 in 9's complement representation is – 100.
  - The sign-and-magnitude value of 3-digit number 170 in 9's complement representation is + 170.
- We can find the 9's Complement representation of a negative number by subtracting the number with $(10^n - 1)$ where n = number of digits in the number.

نشيك العدد مكون من كم خانه 3, 2, 1 و نطرح العدد من 9, 99, 999

- Example: Find the 9's Complement of – 6! The number is only one digit therefore 9 – 6 = 3.
- Example: Solve the following problem 13 – 6 using 9's Complement representation:

هنا اخذنا بالاعتبار عدد خانات الرقم الاكبر 13 في المسائلة

  - 9's Complement: +13 ➡ 13 and – 6 ➡ 99 – 6 = 93
  - Note we use addition:

$$\begin{array}{r} 93 \\ +\ 13 \\ \hline 106 \\ +\ \ 1 \\ \hline 07 \end{array}$$

عدد الخانات 2 لذالك 1 يعتبرفائض يضاف للناتج

  - If the result has more digits than specified, add carry to the result.

10's Complement:

- Based on 9's complement
- Example using 3-digit number
  - Note: 1000 = 999 + 1
  - 9's complement = 999 – value
  - Rewriting 10's complement = 1000 – value = 999 + 1 – value
  - Or: 10's complement = 9's complement + 1

الارقام الموجبة بطريقة 10's complement في حال 3-digit هي من 0 الى 499

Representation	500	***649***	***899***	999	0	***170***	***420***	499
Number represented	-500	***-351***	***-101***	-001	0	***170***	***420***	499

الارقام السلبية بطريقة 10's complement في حال 3-digit هي من999 الى 500

- Example:
  - The sign-and-magnitude value of 3-digit number 999 in 10's complement representation is – 001.
  - The sign-and-magnitude value of 3-digit number 420 in 10's complement representation is + 420.
- We can find the 10's Complement representation of a negative number by subtracting the number with (10^n) where n = number of digits in the number.

نشيك العدد مكون من كم خانه 1, 2, 3 و
نطرح العدد من 10, 100, 1000

- Example: Find the 10's Complement of – 6! The number is only one digit therefore 10 – 6 = 4.
- Example: Solve the following problem 13 – 6 using 10's Complement representation:

هنا اخذنا بالاعتبار عدد خانات الرقم الاكبر 13 في المسائلة

  - 10's Complement: +13 ➡ 13 and – 6 ➡ 100 – 6 = 94
  - Note we use addition:

$$\begin{array}{r} 94 \\ +\ 13 \\ \hline 107 \\ \hline 07 \end{array}$$

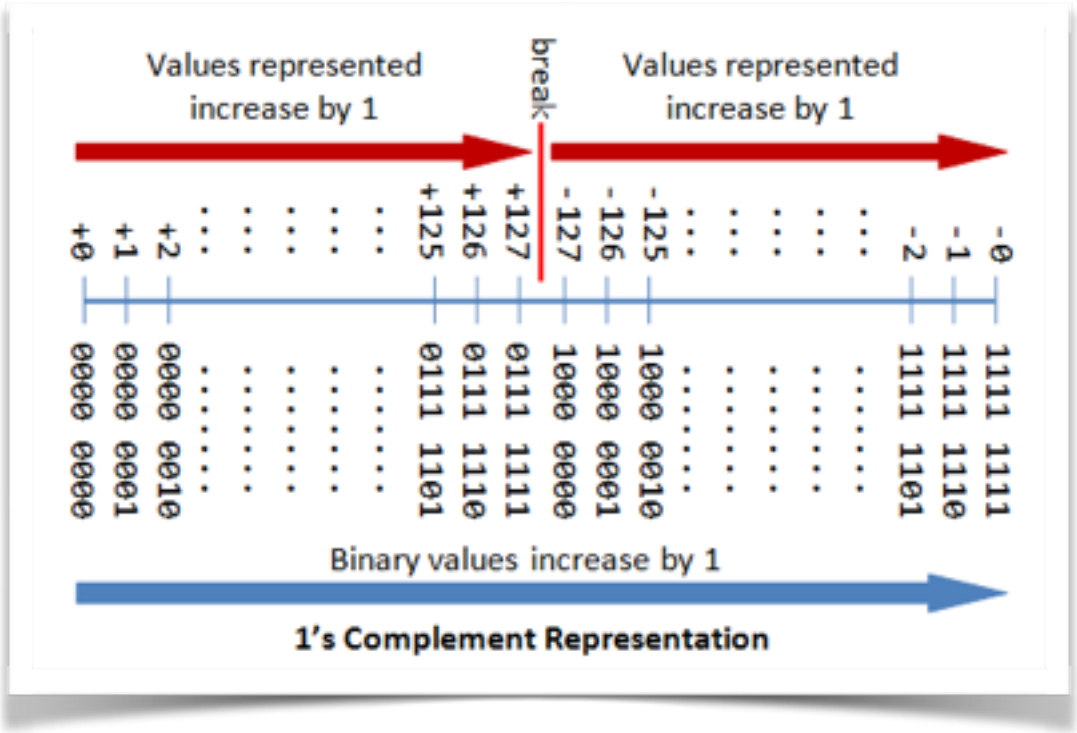
عدد الخانات 2 لذالك 1 يعتبرفائض لا يضاف للناتج

- Example: Find the 10's Complement of 777! Negative number because first digit is 7. The number has 3-digits therefore 1000 – 777 = 223. Signed value = – 223.

1's Binary Complement:

- In the 1's complement representation, a nonnegative number is represented in the same manner as an unsigned number.
- A negative number (−N) is represented by the 1's Binary Complement of the positive number N.
- The 1's Binary Complement of an n-bit number N is obtained by subtracting it from $2^n - 1$
  - First find the binary number of N
  - Inversion: change 1's to 0's and 0's to 1s
  - Numbers beginning with 0 are positive
  - Numbers beginning with 1 are negative
  - 2 values for zero
- Example with 8-bit binary numbers

- نحول الرقم العشري الى رقم الثنائي
- نبدل 0 بـ 1 و الـ 1 بـ 0



- Example: Solve the following problem 13 – 6 using 1's Binary Complement representation:
  - Find the binary number: $(13)_{10}=(1101)_2$ and $(6)_{10}=(0110)_2$
  - 1's Binary Complement: +13 ➡ 1101 and – 6 ➡ 1001
  - Note we use addition:

$$\begin{array}{r} 1\ 1\ 0\ 1 \\ +\ 1\ 0\ 0\ 1 \\ \hline 1\ 0\ 1\ 1\ 0 \\ +\qquad 1 \\ \hline 0\ 1\ 1\ 1 \end{array}$$

  - The result is $(0111)_2 = (7)_{10}$

تضيف اصفار على يسار العدد لتكوين 4 bits او 8 bits

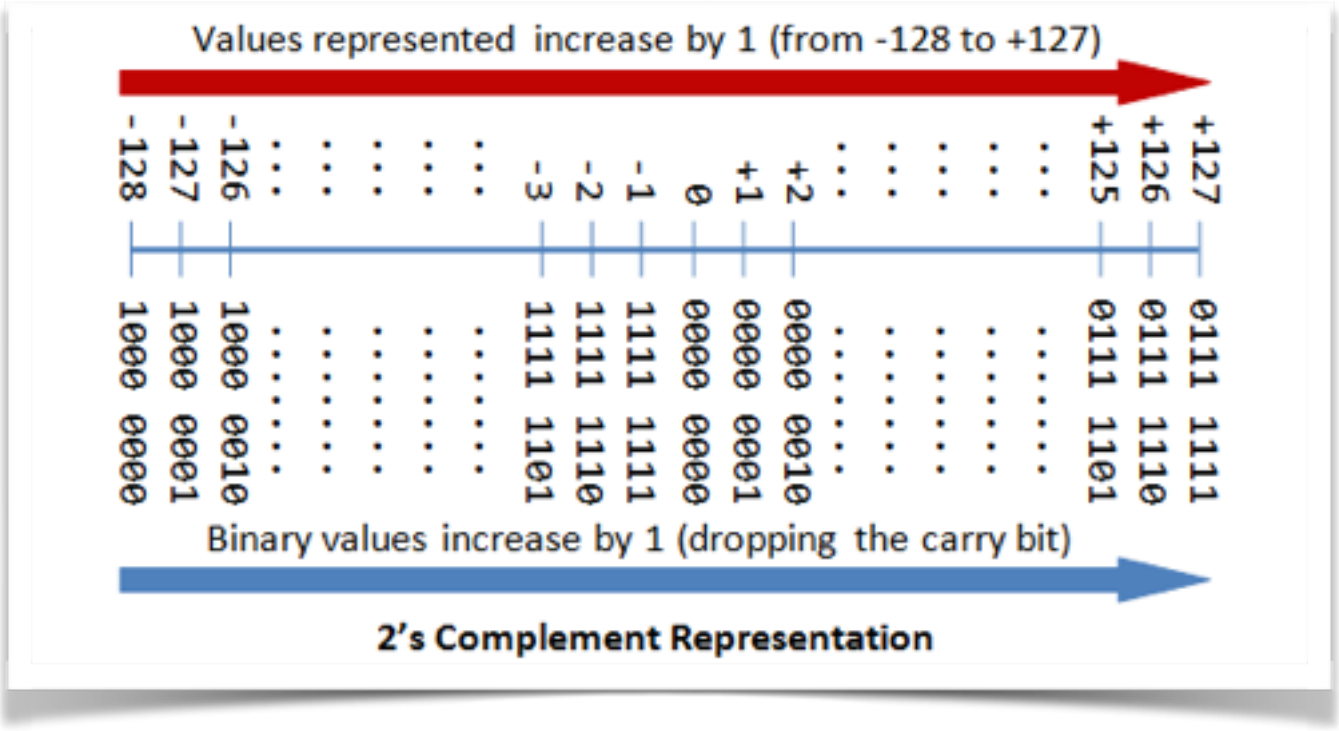
لدينا 4 bits لذالك الـ 1 على اليسار يعتبر فائض يضاف للناتج

2's Binary Complement:

- In the 2's binary complement representation, a nonnegative number is represented in the same manner as an unsigned number.
- A negative number (−N) is represented by 2's complement of the positive number N.
- The 2's Binary Complement of an n-bit number N is obtained by subtracting it from 2^n
  - First find the binary number of N
  - 1's Binary Complement (change 1's to 0's and 0's to 1s)
  - Add 1 to 1's Binary Complement

- نحول الرقم العشري الى رقم الثنائي
- نبدل 0 بـ 1 و الـ 1 بـ 0 (1's complemnt)
- نضيف 1 لـ 1's complemnt

- Example with 8-bit binary numbers



- Example: Solve the following problem 13 – 6 using 2's Binary Complement representation:
  - Find the binary number: $(13)_{10}=(1101)_2$ and $(6)_{10}=(0110)_2$
  - 1's Binary Complement: +13 ➡ 1101 and – 6 ➡ 1001
  - 2's Binary Complement: +13 ➡ 1101 and – 6 ➡ 1010
  - Note we use addition:

$$\begin{array}{r} 1\ 1\ 0\ 1 \\ +\ 1\ 0\ 1\ 0 \\ \hline 1\ 0\ 1\ 1\ 1 \end{array}$$

  - The result is $(0111)_2 = (7)_{10}$

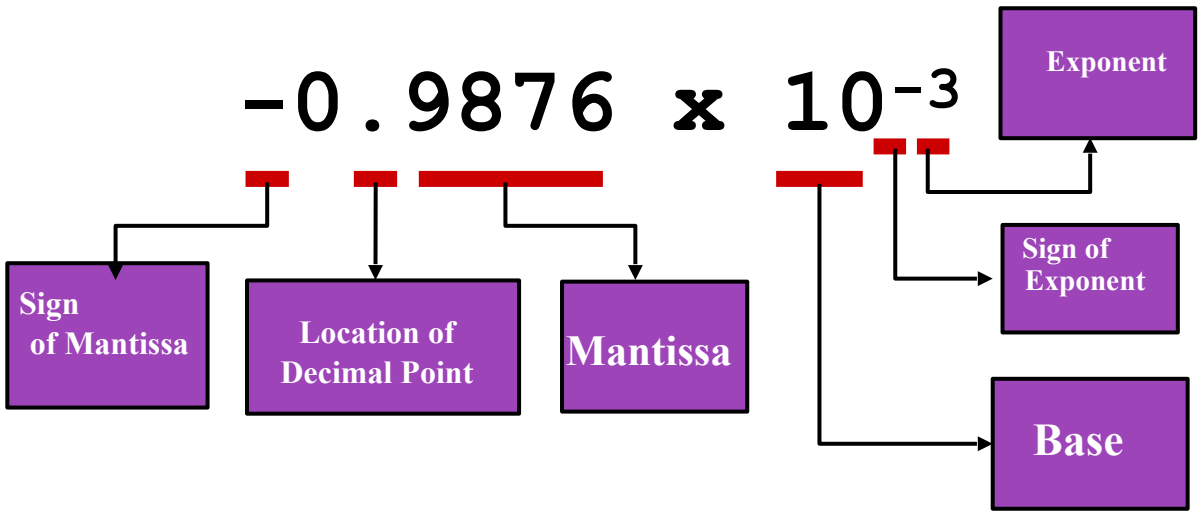
نضيف اصفار على يسار العدد لتكوين 4 bits او 8 bits

1010 = 1001 + 0001

لدينا 5 bits لذالك الـ 1 على اليسار يعتبر فائض لا يضاف للناتج

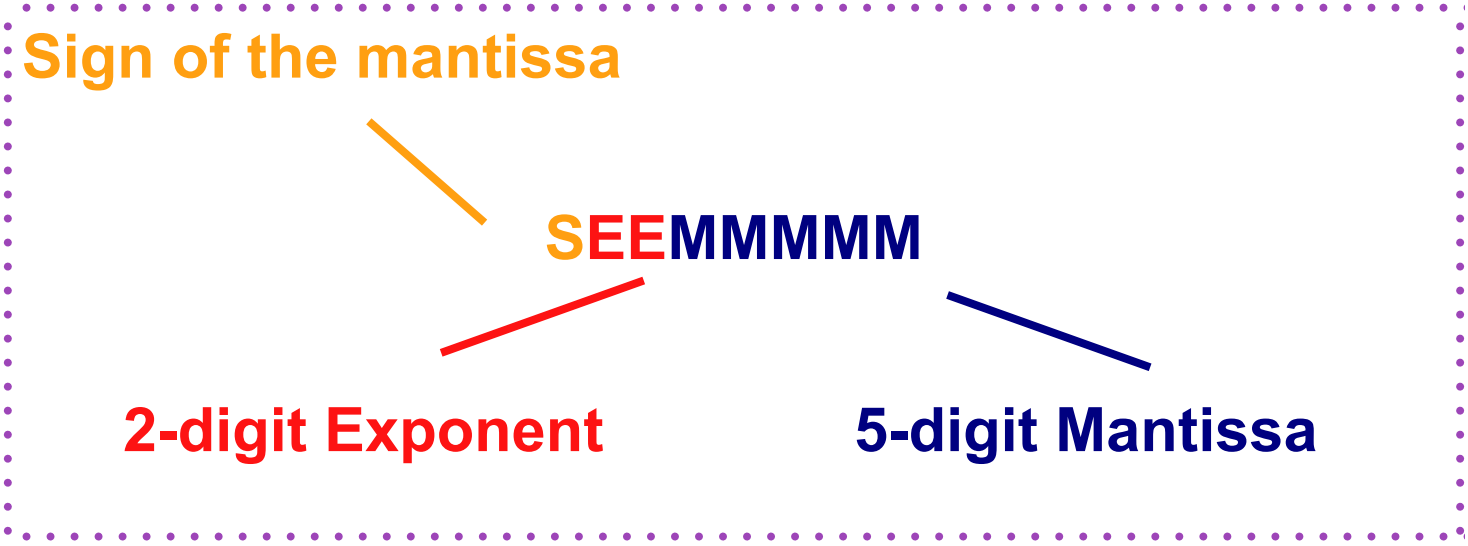
Exponential Notation:

- Also called *scientific notation*
  - 12345
  - 12345×10^{0}
  - 0.12345×10^{5}
  - 123450000×10^{-4}
- 4 specifications required for a number
  1. Sign ("+" in example)
  2. Magnitude or *mantissa* (12345)
  3. Sign of the exponent ("+" in 10^{5})
  4. Magnitude of the exponent (5)
- Plus
  5. Base of the exponent (10)
  6. Location of decimal point (or other base) radix point
- The number -0.0009876 can be represented as -0.9876×10^{-3}



Format Specification:

- Typical representation is using 8 digits: SEEMMMMM, Where:



- Mantissa: sign digit in sign-magnitude format
- Assume decimal point located at beginning of mantissa
- Excess-N notation: Complementary notation
  - Pick middle value as offset where N is the middle value
  - The exponent is represented in Excess-50.
  - That means EE=00=10^{-50}….. EE=48=10^{-2} ,EE=49= 10^{-1} , EE=50=10^{0} ,EE=51=10^{1} , EE=52=10^{2} , EE=53=10^{3}, …… EE=99=10^{49}

Representation	0	49	50	99
Exponent being represented	-50	-1	0	49

– Increasing value +

- SEEMMMMM= (+/-) .MMMMM x 10^{EE}
- Example:
  - 05324657 = +0.24657 x 10^{3} = + 246.57
  - 54810000 = – 0.10000 x 10^{-2} = – 0.001
  - 55555555 = – 0.55555 x 10^{5} = – 55555
  - 04925000 = +0.25000 x 10^{-1} = + 0.025

Addition and Subtraction in SEEMMMMM format:

- Example: Solve the following problem 05199520 + 04967850 using SEEMMMMM format with excess-50 notation and S=0 is (+) and S=5 is (–):
  - Add two floating point numbers: 05199520
    + 04967850
    ———————
  - Align exponents: 0 51 99520
    + 0 51 0067850
    ———————
    0 51 (1)0019850
  - Carry requires right shift 0 52 10019850
  - The result is 05210019850 = + .10019850 x 10^{2}

قبل البداء في الجمع نبدل الأس الاصغر بالأس الأكبر و نضيف اصفار هنا غيرنا 49 الى 51 و اضفنا صفرين بعد الفاصلة لأن 2 =49-51

- We can check our answers by converting the number to decimal:
  - 05199520 = + 0.99521 x 10^{1} = + 9.9520
  - 04967850 = + 0.67850 x 10^{-1} = + 0.067850

نحول الارقام الى شكلها العشري

+10.019850

+0.10019850 x 10^{2}

05210019850

الإجابة صحيحة

Multiplication and Division in SEEMMMMM format:

- Example: Solve the following problem 05199520 x 04967850 using SEEMMMMM format with excess-50 notation and S=0 is (+) and S=5 is (–):
  - Multiply two floating point numbers: 05220000 x 04712500
  - Add exponents, subtract offset: 52 + 47 – 50 = 49
  - Multiply mantissas 0.20000 x 0.12500 = 0.025000000
  - Normalize the result 04825000